

# TEXAS INSTRUMENTS

## S O F T W A R E

# **TI-74 STATISTICS LIBRARY**

## **GUIDEBOOK**



# TEXAS INSTRUMENTS

## TI-74

# STATISTICS

# LIBRARY

## GUIDEBOOK

**Manuals  
developed by:**

The staff of Texas Instruments Instructional  
Communications

**With  
contributions  
by:**

Bruno Didier  
Robert A. Pollan  
S. S. Millwright  
J. H. Puckett  
Robert E. Whitsitt, II

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# How to Use this Guidebook

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**This guidebook describes the contents and use of the TI-74 Statistics Library cartridge. The guidebook is organized to help you use the programs and subprograms in the cartridge.**

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## **Organization of the Guidebook**

The guidebook includes six chapters, an appendix describing the accessible subprograms in the cartridge, and an appendix explaining service and warranty information.

Chapter 1 describes the programs in the cartridge, the general use and care of the cartridge, how to use the Directory program, and how to use an optional printer.

Chapters 2 through 6 provide detailed information about each program in the cartridge. The discussion of each program includes:

- ▶ A brief presentation of general information about the program and the inputs required by the program.
- ▶ Step-by-step instructions for using the program.
- ▶ An application example demonstrating the use of the program.

Appendix A describes the subprograms called by the programs of this library. These subprograms may be useful to you as you develop your own programs. Instead of writing the statements to perform a task, you can call the subprogram that does the task.

Service and warranty information is provided in Appendix B, which is at the end of the guidebook for easy reference.

This chapter introduces you to the TI-74 Statistics Library cartridge and helps you get started using the cartridge.

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# Contents of the Statistics Cartridge

The TI-74 Statistics Library cartridge enables you to perform statistical calculations quickly and easily. The title, filename, and a brief description of each cartridge program are provided below for your reference.

## The Programs

The Statistics cartridge contains the following programs.

Program Title	Filename	Description
Directory	DIR or CONTENTS	Displays the title and filename of each program in the cartridge
Histogram	HIST	Constructs histogram or checks data for goodness-of-fit to a theoretical distribution
Means and Moments	MEAN	Calculates means, moments, skewness, and kurtosis of a sample distribution
One-Way ANOVA	AOV1	Compares means for sample populations with different variances where only one factor varies
Two-Way ANOVA	AOV2	Compares means for sample populations with different variances where more than one factor varies
t-Test: Paired Observations	TTESTP	Determines t-statistic and corresponding number of degrees of freedom for paired samples from two normal distributions
t-Test: Unpaired Observations	TTESTU	Determines t-statistic and degrees of freedom for unpaired samples from two normal distributions

**The Programs  
(Continued)**

Program Title	Filename	Description
Contingency Table Analysis	CTBL	Applies the chi-square test of independence to a contingency table by computing the chi-square statistic and cumulative distribution function of $\chi^2$ , $P(\chi^2)$
Mann-Whitney Rank-Sum Test	RSUM	Uses the Mann-Whitney Rank-Sum test to compute standard deviation, rank mean, and rank variance for two populations with the same distribution
Binomial Distribution	BINOM	Calculates probabilities for n trials of an experiment and computes mean and standard deviation
Chi-Square Distribution	CHI2	Computes the area of the curve to the right of the chi-square value
F-Distribution	FDIS	Computes the area of the curve to the right of the F-statistic
Normal Distribution	NORMAL	Uses standard deviation to compute curve height and area
Poisson Distribution	POISS	Calculates probability of exactly k successes $f(k)$ , k or fewer successes $P(k)$ , and more than k successes $Q(k)$
Student's t-Distribution	STUDENT	Computes $P(t)$ —the area of the curve to the left of the t-statistic

# Using Cartridges with Your TI-74

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Although the cartridges for your TI-74 are durable devices, you should follow these guidelines for installing and caring for cartridges.

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## Caring for the Cartridge

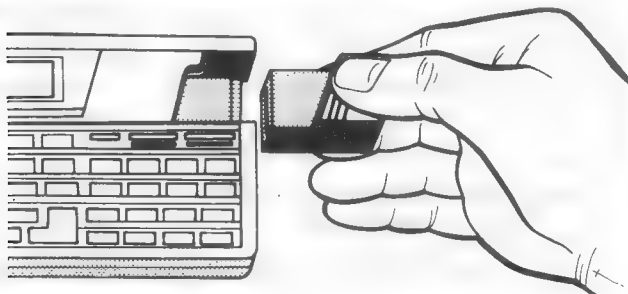
Handle the cartridge with the same care you would give any other piece of electronic equipment. You should:

- ▶ Avoid static electricity. Prior to handling the cartridge, touch a metal object to discharge any static electricity.
- ▶ Store the cartridge in its original container or in the cartridge port on the upper right side of the TI-74.

## Installing a Cartridge

The TI-74 is shipped with a port protector in the cartridge port. The port protector resembles a cartridge and is installed and removed in the same way.

1. Make sure the TI-74 is turned off. Installing a cartridge while the TI-74 is on may result in memory loss.
2. If the port protector or a cartridge is currently in the port, remove it by placing your thumb on the ridged area on top of the cartridge and sliding the cartridge to the right. Store the removed cartridge in its container.
3. Turn the Statistics cartridge so that the ridges are facing upward.
4. Insert the cartridge into the port, small end first.



5. Slide the cartridge to the left until it snaps into place.

You should keep a cartridge or the port protector in the port at all times to prevent the accumulation of dust.

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## Running a Program

To run a cartridge program, follow these general steps:

1. Place the TI-74 in BASIC mode.
2. Type **RUN "filename"** and press **[ENTER]**.

For example, the first program in this library is Histogram, and its filename is "HIST". To execute this program, you type **RUN "HIST"** and press **[ENTER]**.

The program title is displayed for several seconds.

3. Press **[ENTER]** to begin the program immediately, or wait until the program begins automatically.

When running a program, enter all commands and responses exactly as they appear in the step-by-step instructions.

## Pausing During A Program

To pause during the execution of a program, press the **[BREAK]** key.

To resume program execution from the breakpoint, first press **[CLR]** or **[ENTER]** to clear the display. Then type **CON** and press **[ENTER]**.

If you press **[BREAK]** when a prompt is in the display, the prompt may not reappear when you resume the program. However, the prompt still exists and you must respond to the prompt before the TI-74 will proceed to the next step.

**Note:** If you press the **[BREAK]** key and then leave the BASIC mode or turn the TI-74 off, you cannot resume the program with **CON**.

## Cancelling A Program

If you are running a program and wish to cancel it, press the **[BREAK]** key and then press either the **[ENTER]** or **[CLR]** key.

You can then restart the same program, execute a new program, or perform any other TI-74 operation.

# Guidelines for Using the TI-74 Programs

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The TI-74 programs contained in the cartridge provide helpful display prompts during the programs. An explanation of these prompts and a few guidelines for using the programs may be helpful as you get started.

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## Display Prompts

Display prompts are brief sentences that appear in the display to help you enter information into the program. There are three types of display prompts:

- ▶ Entry prompts—these prompts end with a colon and instruct you to enter a value. When applicable, default values appear immediately after the colon. To respond, press **[ENTER]** to accept the current value, or type a new value for the variable and press **[ENTER]**.
- ▶ Question prompts—these prompts end with a “?” and require a yes or no response. To respond, press **[Y]** for yes or **[N]** for no.
- ▶ Message prompts—these prompts explain an operation of a program. Because no information needs to be entered, you can press **[ENTER]** or **[CLR]** to immediately proceed to the next prompt.

## Editing Data in a Program

Many of the programs enable you to edit the elements (data values) that you enter into a program. You have the option of editing all of the data values or selecting specific elements to edit.

**Note:** If you replace a value with a value that has fewer digits, be sure to delete any “leftover” digits that remain in the display. For example, suppose the current value is 389 and you want to change it to 23. If you just type **23**, the value in the display becomes 239. You must delete the remaining digit (the 9) before pressing **[ENTER]**.

To enter a value that has fewer digits than the original value, you can:

- ▶ Press **[CLR]**, type the value, and press **[ENTER]**.
- ▶ Type the value and press **[CTL] [↑] [ENTER]**.
- ▶ Type the value and use the space bar to delete the rest of the entry; then press **[ENTER]**.

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## Entering Data Values

When you enter sample data into a program, the program asks for the maximum number of elements. Specify the total number of data values in your sample, and then be sure to enter that many elements each time you enter a set of data. There are two ways to enter data values:

- Typing the values on the keyboard
- Playing back values recorded on a peripheral device, such as a cassette recorder

**Note:** Some programs do not provide the option to record your data when you enter values from a peripheral device.

## Entering Large Sets of Data

If the number of data values in your sample exceeds the 8K memory capacity, you can still enter the total amount of data into a program. To enter a large amount of data:

1. Divide the data values into two or more sets.
2. When the program prompts you for the number of elements, enter the maximum number of elements **per set**. Follow the display prompts to enter the first set of data.
3. Select the “Input Additional Data” option, and then follow the prompts to enter the next set of data.
4. Continue to enter sets of data with the “Input Additional Data” option. If the final set of data does not have the maximum number of elements, press [E] to end input.

**Note:** The additional data option is presented only after you enter the maximum number of elements. If you intend to add additional elements, be sure to enter the maximum number of elements each time you enter a set of data.

# Using the Directory Program

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The Directory program serves as a table of contents for the programs available with the cartridge. This program also provides a handy reference for program filenames. If you have an optional printer, you may want to print out a copy of the Directory program to use for future reference.

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## The Directory Program

The Directory program displays two names for each program in the cartridge. These include:

- ▶ The title of the program
- ▶ The filename that is used to execute the program

For example, the title for the first program in this library is Histogram and its filename is "HIST".

## Displaying the Names in the Directory

In the Directory program, the four arrow keys display the names in the directory.

- ▶ [**↑**]-Displays the previous title in the directory. If you press this key at the first title in the list, the Directory program prompts: **Exit Program?** Press [**Y**] to exit the Directory program, or press [**N**] to return to the first program title.
- ▶ [**↓**]-Displays the next title in the directory. If you press this key after the last title in the list, the Directory program prompts: **Exit Program?** Press [**Y**] to exit the Directory program, or press [**N**] to return to the last program title.
- ▶ [**→**]-Displays the filename of a program. The TI-74 ignores this key if a filename is already in the display.
- ▶ [**←**]-Displays the title of a program. The TI-74 ignores this key if a title is already in the display.

## Running a Program from the Directory

When the Directory program displays a program title or filename, you can execute that program by pressing the [**RUN**] key. This procedure exits the Directory program and starts the selected program.



**Running the  
Directory  
Program**

To execute the Directory program, type

**RUN "DIR" (or RUN "CONTENTS")**

and press **[ENTER]**. To exit the Directory program, press **[↓]** when the last title or filename is displayed or press **[↑]** when the first title or filename is displayed. Then press **[Y]** when the prompt **Exit Program?** appears in the display.

Step	Display	Procedure/Comment
1	STATISTICS LIBRARY	Wait while the cartridge name is displayed, or press <b>[ENTER]</b> to continue.
-	Use Printer?	a. If you want to print the directory, press <b>[Y]</b> . b. If you do not want to print the directory, press <b>[N]</b> . Go to step 4.
	Enter Device Code:	Type output device code and press <b>[ENTER]</b> . When directory is printed, program stops.
3	program title	Displays title. a. To display filename of same program, press <b>[→]</b> . Go to next step. b. To display title of previous program, press <b>[↑]</b> . Return to step 4. c. To display title of next program, press <b>[↓]</b> . Return to step 4. d. To execute the program, press <b>[RUN]</b> .
	filename	Displays filename. a. To display title of same program, press <b>[←]</b> . Return to step 4. b. To display title of previous program, press <b>[↑]</b> . Return to step 4. c. To display title of next program, press <b>[↓]</b> . Return to step 4. d. To execute the program, press <b>[RUN]</b> .

## Using the Optional Printer

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When you run statistics programs, you may want to use a compatible printer, such as the PC-324 Printer. A printer provides a convenient, permanent record of your programs and data files. Refer to the printer's manual for information on connecting the printer to your TI-74.

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### The Printout

The printout contains all of the information needed to execute a particular program (with the exception of subprograms). The printout includes:

- ▶ Program name
- ▶ Selected options
- ▶ Pertinent input data
- ▶ Results

### Accessing the Printer

To use an optional printer:

1. Respond to the Use Printer? prompt by pressing [Y]. The Enter Device Code: prompt appears in the display.
2. Respond to the Enter Device Code: prompt by typing the appropriate peripheral identification number and pressing [ENTER]. (Refer to your printer's operating manual for the identification number.)

**Note:** When a printed record is produced, the program does not pause while displaying individual results. Instead, the TI-74 prints a continuous list of results until the output is complete.

# Setting the Language of the Prompts

The instructions in this manual show the titles, names, and prompts in English. However, the Statistics cartridge contains a subprogram that lets you also select German or French. You can use this subprogram only when a cartridge program is not running.

## The Languages

Each of the three languages available in the cartridge is identified by a number in the SETLANG subprogram.

Language Number	Language
-----------------	----------

0	English (Default)
---	-------------------

1	German
---	--------

2	French
---	--------

If you run a cartridge program and the prompts are not in the language you prefer, press **[BREAK][CLR]**. Then use the SETLANG subprogram to change the language.

## Setting the Language

The CALL SETLANG(*language-number*) statement specifies the language in which the prompts and messages in the cartridge programs are displayed. For example, the following command sets the language to French.

### CALL SETLANG(2)

**Note:** All system messages are displayed in English, regardless of the language setting.

The Constant Memory feature of the TI-74 retains the language setting when you turn the calculator off and on. The language setting remains in effect until you change it again or until the TI-74 is initialized.

## Yes/No Responses

Changing the language setting from English to German or French also changes the key that indicates a "yes" response. For German, the "yes" response is the **[J]** key. For French, the "yes" response is the **[O]** key. The "no" response is always the **[N]** key, regardless of the language setting.

## Chapter 2: Descriptive Statistics Programs

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The programs in this chapter are designed to help you describe sets of sample data in terms of central tendency and distribution. These programs enable you to construct histograms, compare data to theoretical distributions, and calculate means and moments for grouped and ungrouped data.

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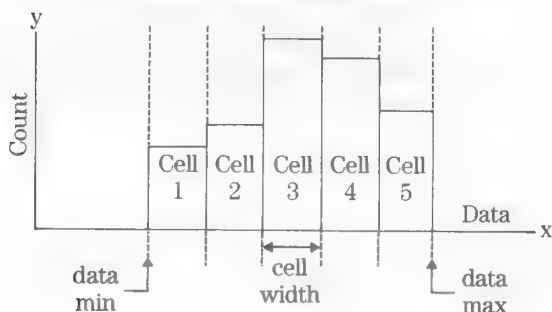
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# The Histogram (HIST) Program

After you collect the data from a sample of a population, you can construct a histogram to help you interpret the set of data values. With the Histogram program, you can construct a histogram from sample data, or you can enter a preconstructed histogram and compare it to a histogram for a theoretical distribution.

## Frequency Histograms

A frequency histogram is a graph that plots the boundaries of each cell against the frequency of occurrence within a cell. Rectangular regions or bars are constructed over each cell with heights proportional to the frequency.



## Uses for the Histogram Program

You can use the Histogram program to:

- ▶ Construct a histogram to determine the number or frequency of counts per cell.
- ▶ Calculate the means and moments of your histogram.
- ▶ Compare the histogram constructed from your sample data points to a histogram for a theoretical distribution.

## Constructing a Histogram

To construct a histogram, you need to provide the program with the following information.

1. Determine the number of cells needed to provide an accurate representation of the sample data. Specify this number as an integer when the prompt Enter No. of Cells appears in the display.

**Note:** Memory requirements for this program are a function of both number of data points and number of cells. Most histograms consist of 20 cells or less, in which case you may have at least 500 data points. To enter a larger data set, refer to "Guidelines for Using the TI-74 Programs," in Chapter 1.

Constructing  
a Histogram  
(Continued)

2. Calculate the width you want each cell to be. When the prompt Enter Cell Width: appears, enter the width.
3. Establish a lower limit (the beginning boundary) for the first cell. Make sure that none of your elements fall on this boundary. For example, if the minimum element is 0, choose a value such as -.5 for the lower limit. Enter the lower limit when the prompt Enter Minimum X Value: appears.

**Note:** If you enter any value less than minimum X or greater than maximum X, the program discards that value and displays the error message \*\*OUT OF RANGE ERROR\*\*.

Comparing  
Theoretical  
Histograms

The Histogram program compares the distribution of sample data to that of a theoretical distribution. The theoretical distributions available with this program are:

- ▶ Normal distribution
- ▶ Binomial distribution
- ▶ Poisson distribution
- ▶ Uniform distribution

The program calculates the expected number of elements in each cell and compiles a chi-square statistic ( $\chi^2$ ). Then, it uses this equation to compare  $\chi^2$  to the theoretical distribution:

$$\chi^2 = \sum_{i=1}^n \frac{(\text{expected count} - \text{observed count})^2}{\text{expected count}}$$

with  $N - 1$  degrees of freedom ( $N$  is the number of cells in your histogram).

### Comparing Theoretical Histograms (Continued)

After completing the goodness-of-fit test, the program calculates  $Q(\chi^2)$ . The observed  $Q(\chi^2)$  value is the probability of rejecting the null hypothesis when it is true (a type I error).

**Note:** If cell widths are not expressed as integers in the Binomial or Poisson distributions, the error message `Value must be an integer` appears in the display.

### Tips for Using the Histogram Program

To obtain the most accurate goodness-of-fit test, follow these guidelines.

- ▶ Collect enough data. A small sample may not be sufficient to make the goodness-of-fit test valid.
- ▶ The expected value of each cell should be greater than or equal to 5.
- ▶ When you use the binomial distribution, the following condition must hold true.

$(\text{Cell Width} \times ((\text{Number of Cells}) - 1) + X_{\min})$  rounded up to an integer  $\leq n$

- ▶ Because the program subtracts the number of degrees of freedom lost from the number of cells, be sure you create a sufficient number of cells.

For example, when you accept the calculated parameters for the normal distribution, three degrees of freedom are lost instead of just one.



## Using the Histogram Program

To execute the Histogram program, type RUN "HIST" and press [ENTER].

Step	Display	Procedure/Comment
	HISTOGRAM	Wait while program name is displayed, or press [ENTER] to continue.
	Use Printer?	a. If you want to use a printer, press [Y]. b. If you do not want to use a printer, press [N]. Go to step 4.
	Enter Device Code:	Type output device code and press [ENTER].
	Theoretical Histogram?	a. To run a goodness-of-fit test, press [Y]. b. To display next option, press [N]. Go to step 6.
	Construct Histogram?	a. To construct histogram, press [Y]. b. To input preconstructed histogram, press [N]. Go to step 8.
	Enter No. Of Elements:	Press [ENTER] to accept current value, or type the maximum number of elements in your sample and press [ENTER].
	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
	Enter No. Of Cells:	Press [ENTER] to accept current value, or type the desired number of cells for histogram and press [ENTER].
	Enter Cell Width:	Press [ENTER] to accept current value, or type the width of cell boundaries and press [ENTER].

## Using the Histogram Program (Continued)

Step	Display	Procedure/Comment
10.	Enter Minimum X Value:	Press <b>[ENTER]</b> to accept current value, or type the lower limit for histogram and press <b>[ENTER]</b> . a. If entering data from peripheral device, go to step 11. b. If constructing histogram from keyboard, go to step 12. c. If entering preconstructed histogram from keyboard, go to step 27.
11.	Enter Device.Filename:	Type device and filename of peripheral device and press <b>[ENTER]</b> . Go to step 14.
12.	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
13.	Enter X(###):	Press <b>[ENTER]</b> to accept current value, or type value for $X_i$ and press <b>[ENTER]</b> . Repeat step until all values are entered. a. If you enter the maximum number of elements (specified in step 6), the next prompt appears automatically. b. If you do not want to enter the maximum number, type <b>E</b> and press <b>[ENTER]</b> to end input.
14.	Edit?	a. To edit data, press <b>[Y]</b> . b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 20. If accepting data entered by peripheral device, go to step 22.
15.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 17.
16.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat this step until all elements are edited. If editing data entered by keyboard, go to step 20. If editing data entered by peripheral device, go to step 22.

Step	Display	Procedure/Comment
17.	Enter Element #:	Type the number of the element to be edited and press <b>[ENTER]</b> .
18.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
19.	Edit More Elements?	a. To edit more elements, press <b>[Y]</b> . Go to step 17. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 20. If accepting data entered by peripheral device, go to step 22.
20.	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 22.
21.	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> .
22.		a. If you input the maximum number of elements, go to step 23. b. If you did not input the maximum number of elements, go to step 24.
23.	Input Additional Data?	a. If you want to input additional elements, press <b>[Y]</b> . Go to step 12. b. If you do not want to input additional elements, press <b>[N]</b> .
24.	See Cell Frequencies?	a. To view frequencies of constructed histogram, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 34.
25.	X Cell(###)=	Displays mean for cell i. Press <b>[ENTER]</b> to proceed.

## Using the Histogram Program (Continued)

Step	Display	Procedure/Comment
26.	f Cell(###)=	Displays frequency for cell i. Press [ENTER] to repeat steps 25 and 26 for each consecutive cell. Press [ENTER] to proceed after last cell. Go to step 34.
27.	Enter f Cell(###):	Press [ENTER] to accept current value, or type frequency for cell i from previously known histogram and press [ENTER].
28.	Edit?	a. To edit data, press [Y]. b. To accept data, press [N]. Go to step 34.
29.	Edit All Data?	a. To edit all data, press [Y]. b. To edit single elements, press [N]. Go to step 31.
30.	Enter f-Cell(###): ####	Press [ENTER] to accept current value, or type correct value and press [ENTER]. Repeat this step until all elements are edited. Go to step 34.
31.	Enter Element #:	Type the number of the element to be edited and press [ENTER].
32.	Enter f-Cell(###): ####	Press [ENTER] to accept current value, or type correct value and press [ENTER].
33.	Edit More Elements?	a. To edit more elements, press [Y]. Go to step 31. b. To accept data, press [N].
34.	Display Statistics?	a. To display means and moments, press [Y]. b. To select next option, press [N]. Go to step 42.
35.	Arithmetic Mean=	Displays arithmetic mean. Press [ENTER] to proceed.

Step	Display	Procedure/Comment
✓	Geometric Mean=	Displays geometric mean. Press [ENTER] to proceed.
✓	Harmonic Mean=	Displays harmonic mean. Press [ENTER] to proceed.
✓	M(2)=	Displays the second moment. Press [ENTER] to proceed.
✓	M(3)=	Displays the third moment. Press [ENTER] to proceed.
✓	M(4)=	Displays the fourth moment. Press [ENTER] to proceed.
✓	No. Of Entries=	Displays number of elements. Press [ENTER] to proceed.
✓		a. If you chose theoretical histogram, go to step 43. b. If you did not choose theoretical histogram, go to step 63.
✓	NORMAL DISTRIBUTION?	a. To use normal distribution for goodness of fit, press [Y]. b. To select next option, press [N]. Go to step 54.
✓	Use Calculated Parameters?	a. To use computed mean and standard deviation, press [Y]. b. To use hypothetical mean and standard deviation, press [N]. Go to step 47.
✓	Mean=	Displays computed mean. Press [ENTER] to proceed.
✓	Standard Dev.=	Displays computed standard deviation. Press [ENTER] to proceed. Go to step 59.

## Using the Histogram Program (Continued)

Step	Display	Procedure/Comment
47.	Enter Mean:	Type hypothetical mean and press <b>[ENTER]</b> .
48.	Enter Standard Dev:	Type hypothetical standard deviation and press <b>[ENTER]</b> . Go to step 59.
49.	BINOMIAL DISTRIBUTION?	a. To use binomial distribution for goodness-of-fit test, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 54.
50.	Enter n:	Type the parameter of binomial and press <b>[ENTER]</b> .
51.	Use Calculated Parameters?	a. To accept calculated probability, press <b>[Y]</b> . b. To enter hypothetical probability, press <b>[N]</b> . Go to step 53.
52.	p=	Displays calculated probability. Press <b>[ENTER]</b> to proceed. Go to step 59.
53.	Enter p:	Type the hypothetical probability and press <b>[ENTER]</b> . Go to step 59.
54.	POISSON DISTRIBUTION?	a. To use Poisson distribution for goodness-of-fit test, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 58.
55.	Use Calculated Parameters?	a. To accept calculated mean, press <b>[Y]</b> . b. To enter hypothetical mean, press <b>[N]</b> . Go to step 57.
56.	m=	Displays calculated mean. Press <b>[ENTER]</b> to proceed. Go to step 59.
57.	Enter m:	Type the hypothetical mean and press <b>[ENTER]</b> . Go to step 59.

Step	Display	Procedure/Comment
63.	UNIFORM DISTRIBUTION?	a. To use uniform distribution for goodness-of-fit test, press <b>[Y]</b> . b. To continue program, press <b>[N]</b> . Go to step 63.
64.	Expected f-Cell(i)=	Displays expected frequency of cell i. Press <b>[ENTER]</b> to display frequency for each cell. Then press <b>[ENTER]</b> to proceed.
65.	df=	Displays degrees of freedom. Press <b>[ENTER]</b> to proceed.
66.	Chi-Square=	Displays chi-square. Press <b>[ENTER]</b> to proceed.
67.	Q(###)=	Displays chi-square statistic. Press <b>[ENTER]</b> to proceed.
68.	Exit Program?	a. To exit program, press <b>[Y]</b> . b. If you chose theoretical histogram, press <b>[N]</b> . Go to step 64. c. If you did not choose theoretical histogram, press <b>[N]</b> . Go to step 4.
69.	Use Same Histogram?	a. To use same histogram, press <b>[Y]</b> . Go to step 43. b. To continue program, press <b>[N]</b> . Go to step 4.



## Application 1: Constructing a Histogram

The example below uses the Histogram program to construct a histogram and calculate the means and moments for a set of data values.

### Example

Construct a histogram from the data below. Use six cells with a width of 2 and a minimum value of 63.95.

68.4, 65.3, 66.5, 69.0, 73.6, 75.9, 69.7, 69.8, 71.0, 70.8, 67.7, 74.4, 69.9, 71.5, 71.1.

To execute the Histogram program, type **RUN "HIST"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	HISTOGRAM	Wait while program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Theoretical Histogram?	Press <b>[N]</b> .
4.	Enter No. of Elements: 0	Type <b>15</b> and press <b>[ENTER]</b> .
5.	Enter Data By Keyboard?	Press <b>[Y]</b> .
6.	Enter No. of Cells: 0	Type <b>6</b> and press <b>[ENTER]</b> .
7.	Enter Cell Width: 0	Type <b>2</b> and press <b>[ENTER]</b> .
8.	Enter Minimum X Value: 0	Type <b>63.95</b> and press <b>[ENTER]</b> .
9.	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
10.	Enter X(1): 0	Type <b>68.4</b> and press <b>[ENTER]</b> .
11.	Enter X(2): 0	Type <b>65.3</b> and press <b>[ENTER]</b> .
12.	Enter X(3): 0	Type <b>66.5</b> and press <b>[ENTER]</b> .

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Step	Display	Procedure/Comment
	Enter X(4): 0	Type <b>69</b> and press <b>[ENTER]</b> .
	Enter X(5): 0	Type <b>73.6</b> and press <b>[ENTER]</b> .
	Enter X(6): 0	Type <b>75.9</b> and press <b>[ENTER]</b> .
	Enter X(7): 0	Type <b>69.7</b> and press <b>[ENTER]</b> .
	Enter X(8): 0	Type <b>69.8</b> and press <b>[ENTER]</b> .
	Enter X(9): 0	Type <b>71</b> and press <b>[ENTER]</b> .
	Enter X(10): 0	Type <b>70.8</b> and press <b>[ENTER]</b> .
	Enter X(11): 0	Type <b>67.7</b> and press <b>[ENTER]</b> .
	Enter X(12): 0	Type <b>74.4</b> and press <b>[ENTER]</b> .
	Enter X(13): 0	Type <b>69.9</b> and press <b>[ENTER]</b> .
	Enter X(14): 0	Type <b>71.5</b> and press <b>[ENTER]</b> .
	Enter X(15): 0	Type <b>71.1</b> and press <b>[ENTER]</b> .
	Edit?	Press <b>[N]</b> .
	Record Data?	Press <b>[N]</b> .
	Input Additional Data?	Press <b>[N]</b> .

## Application 1: Constructing a Histogram (Continued)

Step	Display	Procedure/Comment
28.	See Cell Frequencies?	Press [Y].
29.	X Cell(1)= 64.95 f Cell(1)= 1  X Cell(2)= 66.95 f Cell(2)= 2  X Cell(3)= 68.95 f Cell(3)= 5  X Cell(4)= 70.95 f Cell(4)= 4  X Cell(5)= 72.95 f Cell(5)= 1  X Cell(6)= 74.95 f Cell(6)= 2	Displays all requested values. Proceed after each output by pressing [ENTER].
30.	Display Statistics?	Press [Y].
31.	Arithmetic Mean= 70.01666667 Geometric Mean= 69.96411141 Harmonic Mean= 69.9118111  M(2)= 7.395555556 M(3)= 4.98725902 M(4)= 140.272825  No. Of Entries= 15	Displays all requested values. Proceed after each output by pressing [ENTER].
32.	Exit Program?	Press [Y].

## Application 2: Fitting Data to the Binomial Distribution

You can also use the Histogram program to fit data to a theoretical distribution. By comparing the statistical calculations for your sample with an existing distribution to determine "goodness of fit," you can make generalizations about your total population.

### Example

Five coins were flipped simultaneously one hundred times. The number of heads obtained during each flip was counted and recorded below.

Heads	Observations	Heads	Observations
0	5	3	30
1	14	4	15
2	32	5	4

Use the histogram program to see how well this data fits a binomial distribution.

To execute the Histogram program, type **RUN "HIST"** and press **[ENTER]**.

Step	Display	Procedure/Comment
	HISTOGRAM	Wait while program name is displayed, or press <b>[ENTER]</b> to continue.
	Use Printer?	Press <b>[N]</b> .
	Theoretical Histogram?	Press <b>[Y]</b> .
	Construct Histogram?	Press <b>[N]</b> .
	Enter No. Of Cells: 0	Type <b>6</b> and press <b>[ENTER]</b> .
	Enter Cell Width: 0	Type <b>1</b> and press <b>[ENTER]</b> .
	Enter Minimum X Value: 0	Type <b>-.5</b> and press <b>[ENTER]</b> .
	Enter f-Cell(1): 0	Type <b>5</b> and press <b>[ENTER]</b> .

## Application 2: Fitting Data to the Binomial Distribution (Cont.)

Step	Display	Procedure/Comment
9.	Enter f-Cell(2): 0	Type <b>14</b> and press <b>[ENTER]</b> .
10.	Enter f-Cell(3): 0	Type <b>32</b> and press <b>[ENTER]</b> .
11.	Enter f-Cell(4): 0	Type <b>30</b> and press <b>[ENTER]</b> .
12.	Enter f-Cell(5): 0	Type <b>15</b> and press <b>[ENTER]</b> .
13.	Enter f-Cell(6): 0	Type <b>4</b> and press <b>[ENTER]</b> .
14.	Edit?	Press <b>[N]</b> .
15.	Display Statistics?	Press <b>[Y]</b> .
16.	Arithmetic Mean= 2.48 Geometric Mean= 2.278944632 Harmonic Mean= 2.244668911  M(2)= 1.3696 M(3)= -.042816 M(4)= 5.01578752  No. Of Entries= 100	Displays all requested values. Proceed after each output by pressing <b>[ENTER]</b> .
17.	NORMAL DISTRIBUTION?	Press <b>[N]</b> .
18.	BINOMIAL DISTRIBUTION?	Press <b>[Y]</b> .
19.	Enter n: 0	Type <b>5</b> and press <b>[ENTER]</b> .
20.	Use Calculated Parameters?	Press <b>[N]</b> .
21.	Enter p: 0	Type <b>.5</b> and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
1	Expected f-Cell(1)= 3.125 Expected f-Cell(2)= 15.625 Expected f-Cell(3)= 31.25 Expected f-Cell(4)= 31.25 Expected f-Cell(5)= 15.625 Expected f-Cell(6)= 3.125  df= 5 Chi-Square= 1.632 Q(1.632)= .8973528953	Displays all requested values. Proceed after each output by pressing <b>[ENTER]</b> .
2	Exit Program?	Press <b>[Y]</b> .

**Summary**

The program calculates a  $Q(\chi^2)$  value of 0.8973528953 for the binomial distribution. If you reject the hypothesis, you risk a 89.7% chance of rejecting the null hypothesis when it is true (a type I error).

# The Means and Moments (MEAN) Program

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The means and moments of a sample can accurately describe the shape of its distribution. For a given set of input data ( $x_1, x_2, \dots, x_n$ ) with associated frequencies ( $f_1, f_2, \dots, f_n$ ), this program calculates the means, moments, skewness, and kurtosis of a sample distribution.

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## The Sample Data

Before analyzing sample data with the Means and Moments program, you must decide whether the data is grouped or ungrouped.

- ▶ If  $f_i = 1$  for any  $i$ , the data set is ungrouped.
- ▶ If  $f_i \neq 1$  for any  $i$ , the data set is grouped.

When you specify the number of elements in a sample, you can enter up to that amount when you enter the initial elements and when you enter additional elements. For example, if you respond to the prompt Enter No. Of Elements: with 8, you can enter 8 elements initially and then enter additional elements 8 at a time.

## Memory Capacity

With the standard 8K byte memory capacity, you may enter at least 500 elements for grouped data or 1000 elements for ungrouped data.

## Arithmetic Mean

The arithmetic mean is the average value of the sample data.

- ▶ For ungrouped data, it is the simple arithmetic mean.
- ▶ For grouped data, it is the weighted arithmetic mean.

The arithmetic mean is defined as:

$$\bar{x}_a = (1/N) \sum_{i=1}^n f_i x_i$$

where

$$N = \sum_{i=1}^n f_i$$

## Geometric Mean

The geometric mean is defined as:

$$\bar{x}_g = (x_1^{f_1} \times x_2^{f_2} \times \dots \times x_k^{f_k})^{1/N}$$

where  $N = \sum f_i$  and  $k$  = the last element in the data set.



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**Harmonic  
Mean**

The harmonic mean is defined as:

$$\bar{x}_h = N \div \sum_{i=1}^n f_i/x_i$$

**Second Moment:  
variance**

The second moment (variance) is the mean of squared deviations of the sample data from  $\bar{x}$ . This moment is used to measure the variability or dispersion of a population. The variance is defined as:

$$m_2 = 1/N \sum_{i=1}^n f_i (x_i - \bar{x})^2$$

**Third Moment:  
Skewness**

The third moment (skewness) is defined as:

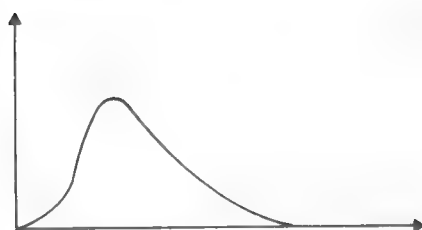
$$m_3 = 1/N \sum_{i=1}^n f_i (x_i - \bar{x})^3$$

Negative and positive deviations cancel each other out because all deviations in the equation for  $m_3$  are cubed. Therefore,  $m_3$  equals zero when the distribution is symmetric about its mean. A distribution is said to be:

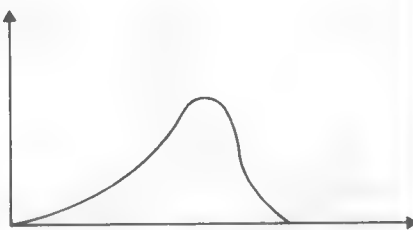
- Right (positively) skewed when  $m_3$  is positive.
- Left (negatively) skewed when  $m_3$  is negative.

### Third Moment: Skewness (Continued)

The following graphs display two skewed curves.



Positively (Right) Skewed



Negatively (Left) Skewed

### Skewness Coefficient

The equation for finding the skewness coefficient is:

$$\text{Skewness Coefficient} = m_3 / (m_2)^{3/2}$$

### The Fourth Moment

The fourth moment helps you interpret the flatness or peakedness of a distribution curve and is defined as:

$$m_4 = 1/N \sum_{i=1}^n f_i(x_i - \bar{x})^4$$

The kurtosis of distribution is determined with the equation

$$\text{Kurtosis coefficient} = m_4 / (m_2)^2$$

The kurtosis of a normal distribution is 3. For values less than 3, the curve flattens out. For values greater than 3, the curve becomes more peaked.

### Reference

*Standard Mathematical Tables*, The Chemical Rubber Co., 1971, pp. 554-559

## Using the Means and Moments Program

To execute the Means and Moments program, type RUN "MEAN" and press [ENTER].

Step	Display	Procedure/Comment
	MEANS & MOMENTS	Wait while program name is displayed, or press [ENTER] to continue.
	Use Printer?	a. If you want to use a printer, press [Y]. b. If you do not want to use a printer, press [N]. Go to step 4.
	Enter Device Code:	Type output device code and press [ENTER].
	Enter No. Of Elements:	Type the number of elements in the sample and press [ENTER].
	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
	Is Data Grouped?	a. To select grouped data, press [Y]. If entering data by keyboard, go to step 8; if entering data by peripheral device, go to step 7. b. To select ungrouped data, press [N]. If entering data by keyboard, go to step 8; if entering data by peripheral device, go to step 7.
	Enter Device.Filename:	Type the device number and filename on which data is stored and press [ENTER]. Go to step 11.
	To End Input Enter 'E'	Press [ENTER] to proceed.
	Enter X(###):	Press [ENTER] to accept current value, or type the value for $X_i$ and press [ENTER]. a. If you enter the maximum number of elements and data set is ungrouped, repeat this step until all values are entered. Go to step 11. b. If the data set is grouped, go to step 10. c. If you do not want enter the maximum number of elements, type E and press [ENTER] to end input. Go to step 11.

## Using the Means and Moments Program (Continued)

Step	Display	Procedure/Comment
10.	Enter f(###):	Press <b>[ENTER]</b> to accept current value, or type the frequency for cell i and press <b>[ENTER]</b> . Repeat steps 9 and 10 until all values are entered. Go to step 11.
11.	Edit?	<ol style="list-style-type: none"><li>To edit data, press <b>[Y]</b>.</li><li>To accept data, press <b>[N]</b>. If accepting data entered by keyboard, go to step 19. If accepting data entered by peripheral device, go to step 21.</li></ol>
12.	Edit All Data?	<ol style="list-style-type: none"><li>To edit all data, press <b>[Y]</b>.</li><li>To edit single elements press <b>[N]</b>. Go to step 15.</li></ol>
13.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . <ol style="list-style-type: none"><li>If data is grouped, go to step 14.</li><li>If data is ungrouped and entered by keyboard, repeat this step until all values are edited. Go to step 19.</li><li>If data is ungrouped and entered by peripheral device, repeat this step until all values are edited. Go to step 21.</li></ol>
14.	Enter f(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat steps 13 and 14 until all values are edited. If editing data entered by keyboard, go to step 19. If editing data entered by peripheral device, go to step 21.
15.	Enter Element #:	Type the number of the element you want to edit and press <b>[ENTER]</b> .
16.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> . <ol style="list-style-type: none"><li>If data is ungrouped, go to step 18.</li><li>If data is grouped, go to next step.</li></ol>

Step	Display	Procedure/Comment
	Enter f(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
	Edit More Elements?	a. To edit more elements, press <b>[Y]</b> . Go to step 15. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 19. If accepting data entered by peripheral device, go to step 21.
	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . If the maximum number of elements was entered, go to step 21. If less than the maximum number of elements was entered, go to step 22.
	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> . Go to step 22.
	Input Additional Data?	a. To input additional data (up to the specified maximum number of elements), press <b>[Y]</b> . Go to step 8. b. To select next option, press <b>[N]</b> .
	Display Means?	a. To display means, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 26.
	Arithmetic Mean=	Displays arithmetic mean. Press <b>[ENTER]</b> to proceed.
	Harmonic Mean=	Displays harmonic mean. Press <b>[ENTER]</b> to proceed.
	Geometric Mean=	Displays geometric mean. Press <b>[ENTER]</b> to proceed.

## Using the Means and Moments Program (Continued)

Step	Display	Procedure/Comment
26.	Display Moments?	a. To display moments, press [Y]. b. To select next option, press [N]. Go to step 30.
27.	M(2)=	Displays the second moment. Press [ENTER] to proceed.
28.	M(3)=	Displays the third moment. Press [ENTER] to proceed.
29.	M(4)=	Displays the fourth moment. Press [ENTER] to proceed.
30.	Display Statistics?	a. To display statistics, press [Y]. b. To select next option, press [N]. Go to step 34.
31.	Skew=	Displays the skew. Press [ENTER] to proceed.
32.	Kurtosis=	Displays the kurtosis. Press [ENTER] to proceed.
33.	No. Of Entries=	Displays the number of elements. Press [ENTER] to proceed.
34.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

## Application 1: Comparing Two Sets of Ungrouped Data

This example compares the means and moments of two sets of ungrouped data.

### Example

The following lists show the respective heights in inches of a random sample of 10 men over age 45 and their adult sons.

Fathers: 67.2, 65.0, 68.3, 69.9, 66.3, 69.7, 69.5, 72.9, 70.2, 74.1.

Sons: 68.4, 65.3, 66.5, 69.0, 73.6, 75.9, 69.7, 69.8, 71.0, 70.8, 67.7, 74.4, 69.9, 71.5, 71.1.

Compare the distribution of the heights of the fathers against that of the sons.

To execute the Means & Moments program, type **RUN "MEAN"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	MEANS & MOMENTS	Wait while program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
	Enter No. Of Elements: 0	Type <b>10</b> and press <b>[ENTER]</b> .
	Enter Data By Keyboard?	Press <b>[Y]</b> .
	Is Data Grouped?	Press <b>[N]</b> .
	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
	Enter X(1): 0	Type <b>67.2</b> and press <b>[ENTER]</b> .
	Enter X(2): 0	Type <b>65</b> and press <b>[ENTER]</b> .
	Enter X(3): 0	Type <b>68.3</b> and press <b>[ENTER]</b> .

## Application 1: Comparing Two Sets of Ungrouped Data (Continued)

Step	Display	Procedure/Comment
10.	Enter X(4): 0	Type <b>69.9</b> and press <b>[ENTER]</b> .
11.	Enter X(5): 0	Type <b>66.3</b> and press <b>[ENTER]</b> .
12.	Enter X(6): 0	Type <b>69.7</b> and press <b>[ENTER]</b> .
13.	Enter X(7): 0	Type <b>69.5</b> and press <b>[ENTER]</b> .
14.	Enter X(8): 0	Type <b>72.9</b> and press <b>[ENTER]</b> .
15.	Enter X(9): 0	Type <b>70.2</b> and press <b>[ENTER]</b> .
16.	Enter X(10): 0	Type <b>74.1</b> and press <b>[ENTER]</b> .
17.	Edit?	a. If you entered the data correctly, press <b>[N]</b> . b. If you would like to recheck your data, press <b>[Y]</b> .
18.	Record Data?	Press <b>[N]</b> .
19.	Input Additional Data?	Press <b>[N]</b> .
20.	Display Means?	Press <b>[Y]</b> .
21.	Arithmetic Mean= 69.31 Harmonic Mean= 69.20924469 Geometric Mean= 69.25951514	Displays each mean. Proceed after each output by pressing <b>[ENTER]</b> .
22.	Display Moments?	Press <b>[Y]</b> .
23.	M(2)= 7.0269 M(3)= 3.938892 M(4)= 114.132659	Displays moments. Proceed after each output by pressing <b>[ENTER]</b> .
24.	Display Statistics?	Press <b>[Y]</b> .
25.	Skew= .2114600903 Kurtosis= 2.311438747 No. Of Entries= 10	Displays each statistic. Proceed after each output by pressing <b>[ENTER]</b> .



Step	Display	Procedure/Comment
26.	Exit Program?	Press [N].
27.	Enter No. Of Elements: 10	Type 15 and press [ENTER].
28.	Enter Data By Keyboard?	Press [Y].
29.	Is Data Grouped?	Press [N].
30.	To End Input Enter 'E'	Press [ENTER] to proceed.
31.	Enter X(1): 0	Type 68.4 and press [ENTER].
32.	Enter X(2): 0	Type 65.3 and press [ENTER].
33.	Enter X(3): 0	Type 66.5 and press [ENTER].
34.	Enter X(4): 0	Type 69 and press [ENTER].
35.	Enter X(5): 0	Type 73.6 and press [ENTER].
36.	Enter X(6): 0	Type 75.9 and press [ENTER].
37.	Enter X(7): 0	Type 69.7 and press [ENTER].
38.	Enter X(8): 0	Type 69.8 and press [ENTER].
39.	Enter X(9): 0	Type 71 and press [ENTER].
40.	Enter X(10): 0	Type 70.8 and press [ENTER].
41.	Enter X(11): 0	Type 67.7 and press [ENTER].
42.	Enter X(12): 0	Type 74.4 and press [ENTER].
43.	Enter X(13): 0	Type 69.9 and press [ENTER].
44.	Enter X(14): 0	Type 71.5 and press [ENTER].
45.	Enter X(15): 0	Type 71.1 and press [ENTER].

## Application 1: Comparing Two Sets of Ungrouped Data (Continued)

Step	Display	Procedure/Comment
46.	Edit?	Press [N].
47.	Record Data?	Press [N].
48.	Input Additional Data?	Press [N].
49.	Display Means?	Press [Y].
50.	Arithmetic Mean= 70.30666667 Harmonic Mean= 70.19915809 Geometric Mean= 70.25279208	Displays each mean. Proceed after each output by pressing [ENTER].
51.	Display Moments?	Press [Y].
52.	M(2)= 7.609955556 M(3)= 4.93266703 M(4)= 152.04781	Displays moments. Proceed after each output by pressing [ENTER].
53.	Display Statistics?	Press [Y].
54.	Skew= .2349679456 Kurtosis= 2.625523613 No. Of Entries= 15	Displays each statistic. Proceed after each output by pressing [ENTER].
55.	Exit Program?	Press [Y].

## Application 2: Using Grouped Data

When large sets of data are involved, you may find it easier to group your data before entering it. Using part of the data from the previous example, this example computes the means and moments of the sons' heights after grouping the data.

### Example 2

The sample heights of the sons from the last example are grouped below.

Range	64<X<66	66<X<68	68<X<70	70<X<72	72<X<74	74<X<76
Frequency	1	2	5	4	1	2
Assigned value	65	67	69	71	73	75

To execute the Means and Moments program, type **RUN** "MEAN" and press **[ENTER]**.

Step	Display	Procedure/Comment
	MEANS & MOMENTS	Wait while program name is displayed, or press <b>[ENTER]</b> to continue.
2	Use Printer?	Press <b>[N]</b> .
	Enter No. Of Elements: 0	Type <b>10</b> and press <b>[ENTER]</b> .
4	Enter Data By Keyboard?	Press <b>[Y]</b> .
5	Is Data Grouped?	Press <b>[Y]</b> .
	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
7	Enter X(1): 0	Type <b>65</b> and press <b>[ENTER]</b> .
	Enter f(1): 0	Type <b>1</b> and press <b>[ENTER]</b> .
	Enter X(2): 0	Type <b>67</b> and press <b>[ENTER]</b> .
	Enter f(2): 0	Type <b>2</b> and press <b>[ENTER]</b> .
	Enter X(3): 0	Type <b>69</b> and press <b>[ENTER]</b> .
	Enter f(3): 0	Type <b>5</b> and press <b>[ENTER]</b> .

## Application 2: Using Grouped Data (Continued)

Step	Display	Procedure/Comment
13.	Enter X(4): 0	Type <b>71</b> and press <b>[ENTER]</b> .
14.	Enter f(4): 0	Type <b>4</b> and press <b>[ENTER]</b> .
15.	Enter X(5): 0	Type <b>73</b> and press <b>[ENTER]</b> .
16.	Enter f(5): 0	Type <b>1</b> and press <b>[ENTER]</b> .
17.	Enter X(6): 0	Type <b>75</b> and press <b>[ENTER]</b> .
18.	Enter f(6): 0	Type <b>2</b> and press <b>[ENTER]</b> .
19.	Enter X(7): 0	Type <b>E</b> and press <b>[ENTER]</b> to end data input.
20.	Edit?	Press <b>[N]</b> .
21.	Record Data?	Press <b>[N]</b> .
22.	Display Means?	Press <b>[Y]</b> .
23.	Arithmetic Mean= 70.06666667 Harmonic Mean= 69.96188555 Geometric Mean= 70.01414879	Displays each mean. Proceed after each output by pressing <b>[ENTER]</b> .
24.	Display Moments?	Press <b>[Y]</b> .
25.	M(2)= 7.395555556 M(3)= 4.98725903 M(4)= 140.272828	Displays each moment. Proceed after each output by pressing <b>[ENTER]</b> .
26.	Display Statistics?	Press <b>[Y]</b> .
27.	Skew= .2479737556 Kurtosis= 2.564672677 No. Of Elements= 15	Displays each statistic. Proceed after each output by pressing <b>[ENTER]</b> .
28.	Exit Program?	Press <b>[Y]</b> .

## Chapter 3: Analysis of Variance Programs

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This chapter describes the Analysis of Variance programs, which test the hypothesis that a number of populations have the same mean. To use these programs with your data, you must be able to assume that the distributions are normal and the population variances are approximately equal.

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# The One-Way Analysis of Variance (AOV1) Program

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The One-Way Analysis of Variance program is designed to test the hypothesis that more than two treatments have the same effect.

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## Testing the Hypotheses

The One-Way ANOVA test is used to analyze several sample populations where only one factor is considered. The program computes the degrees of freedom, sum of squares, mean square, and the F-statistic. This information is needed to use the F-distribution program (presented later) to test the null hypothesis,  $H_0 = \mu_1 = \mu_2 = \dots \mu_n$ .

Two assumptions concerning the analysis of variance are:

1. The populations are normally distributed.
2. The population variances are approximately equal.

## Analysis of Variance

Population samples used in one-way analysis of variance are called treatment groups. Each treatment group  $i$  ( $i = 1, 2, \dots, K$ ) consists of  $n_i$  observations, and each observation in the treatment group  $i$  is designated as  $x_{ij}$  ( $j = 1, 2, \dots, n_i$ ). The different groups need not have the same number of observations.

You can organize the data from your population samples into a table with columns and rows. The rows represent the treatments (population samples) and the columns represent the repetitions or data values.

Each data value is labeled as  $x_{ij}$ , where:

- $i$  is the treatment (row number) for  $i = 1, 2, \dots, K$
- $j$  is the observation (column number) for  $j = 1, 2, \dots, n_i$

This notation enables you to easily locate a data value on an ANOVA table. For example, the data value  $x_{23}$  refers to the data value in the third column of the second row, or the third measurement in the second treatment.

**Analysis of  
Variance  
(Continued)**

The following table shows a standard ANOVA chart.

Level	Replicates										$T_{i.}$
	1	2	3	4	5	6	7	8	9	10	
1	2	6	4	13	5	8	4	6			48
2	6	4	4	1	8	2	12	1	5	2	45
3	2	1	3	3	1	7	1	4	2		24
											$T_{..} = 117$

where  $T$  Specifies a total.

- Specifies composition of the sum.

Therefore,  $T_{i.}$  indicates the total of the  $i$ th row, and the dot indicates that the total includes all column values in that row.  $T_{..}$  indicates the grand total of all the observations.

The formula for finding the total sum of squares is

$$SS\sum = \sum_{i=1}^K \sum_{j=1}^{n_i} x_{ij}^2 - T_{..}^2/N$$

The row totals ( $T_{i.}$ ) are used to calculate sum of squares for treatment rows (SST) by the following formula.

$$SST = \sum_{i=1}^K T_{i.}^2/n_i - T_{..}^2/N$$

Finally, the sum of squares for error (SSE) may be calculated from the following formula.

$$SSE = SS\sum - SST$$

Dividing SST and SSE by their respective degrees of freedom (df) defines the variance terms for the F test.

$$df \sum = N - 1$$

$$df T = K - 1$$

$$df E = N - K$$

## Method Used in the Program

Given the above terms and the preceding table, the ANOVA table for those values can be constructed. The mean square is the total sum of squares divided by df. The two mean square formulas used in this program are provided below and are followed by the ANOVA table constructed from the values in the preceding standard ANOVA table.

$$MST = SST/df T$$

$$MSE = SSE/df E$$

Source	Sum of Squares	df	Mean Square
Rows	47.5	2	23.75 (MSR)
Error	216.5	24	9.02 (MSE)
Total	264	26	

The F-statistic  $F = MST/MSE$  with df T in the numerator and df E in the denominator is then calculated by the program. You can use this data and the F-distribution program to test the hypothesis that the means of your populations are equal.

## Reference

*Statistics*, Rickmers & Todd, McGraw-Hill, 1967, pp. 162-166.



## Using the One-Way ANOVA Program

To execute the One-Way ANOVA program, type RUN "AOV1" and press [ENTER].

Step	Display	Procedure/Comment
	ONE WAY ANOVA	Wait while the program name is displayed, or press [ENTER] to continue.
	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
	Enter Device Code:	Type output device code and press [ENTER].
	Enter data by keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
	Enter Treatment Number: #	Press [ENTER] to accept current value, or type the number of the treatment and press [ENTER].
	Enter No. Of Elements:	Type the number of replicates for the treatment and press [ENTER]. a. If you are entering data from keyboard, go to step 8. b. If you are entering data from peripheral device, go to step 7.
	Enter Device.Filename:	Type the device and filename on which data is stored and press [ENTER]. Go to step 10.
	To End Input Enter 'E'	Press [ENTER] to proceed.
	Enter X(##,##):	a. Press [ENTER] to accept current value, or type the value for $X_{ij}$ and press [ENTER]. Repeat this step until all values are entered. b. If you do not want to enter the total number of elements (specified in step 6), type E and press [ENTER] to indicate end of data input.

## Using the One-Way ANOVA Program (Continued)

Step	Display	Procedure/Comment
10.	Edit?	a. To edit data, press <b>[Y]</b> . b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 16. If accepting data entered by peripheral device, go to step 18.
11.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 13.
12.	Enter X(##,##): #####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat this step until all values are edited. If editing data entered by keyboard, go to step 16. If editing data entered by peripheral device, go to step 18.
13.	Enter Column Number:	Type the number of the column you want to edit and press <b>[ENTER]</b> .
14.	Enter X(##,##): #####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
15.	Edit More Elements?	a. To edit more values, press <b>[Y]</b> . Go to step 13. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 16. If accepting data entered by peripheral device, go to step 18.
16.	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 18.
17.	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> .
18.	More Treatments?	a. To enter more treatments, press <b>[Y]</b> . Go to step 5. b. To continue the program, press <b>[N]</b> .

Step	Display	Procedure/Comment
19.	SS(TOTAL)=	Displays total sum of squares. Press <b>[ENTER]</b> to proceed.
20.	df(TOTAL)=	Displays total degrees of freedom. Press <b>[ENTER]</b> to proceed.
21.	SST=	Displays sum of squares treatment. Press <b>[ENTER]</b> to proceed.
22.	dfT=	Displays degrees of freedom treatment. Press <b>[ENTER]</b> to proceed.
23.	MST=	Displays mean square treatment. Press <b>[ENTER]</b> to proceed.
24.	SSE=	Displays sum of squares error. Press <b>[ENTER]</b> to proceed.
25.	dfE=	Displays degrees of freedom error. Press <b>[ENTER]</b> to proceed.
26.	MSE=	Displays mean square error. Press <b>[ENTER]</b> to proceed.
27.	F=	Displays F statistic. Press <b>[ENTER]</b> to proceed.
28.	Exit Program?	a. To exit program, press <b>[Y]</b> . b. To continue program, press <b>[N]</b> . Go to step 4.

## Application: Comparing the Effectiveness of Drugs

In this example, you use the One-Way ANOVA program to organize the data and find the information needed to compare the means for several populations with the same distribution.

### Example

Extend the example from the "Unpaired t-test" section to compare the effects of three drugs used for the relief of pain. Assume that three groups of patients with the same illness are chosen for the experiment and are given drugs x, y, and z respectively. The resulting number of hours of relief for each patient is given below.

		Repetition
Treatment	Drug x	2, 6, 4, 13, 5, 8, 4, 6
	Drug y	6, 4, 4, 1, 8, 2, 12, 1, 5, 2
	Drug z	2, 1, 3, 3, 1, 7, 1, 4, 2

Test the hypothesis

$H_0$ : All of the drugs relieve pain for the same amount of time  
( $\bar{x} = \bar{y} = \bar{z}$ )

against the alternate hypothesis

$H_1$ : All of the drugs do not relieve pain for the same amount of time.

Since the t-test can only be used for comparing two sample populations, use the one-way analysis of variance to perform this test.

To execute the One-Way ANOVA program, type **RUN "AOV1"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	ONE WAY ANOVA	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter data by Keyboard?	Press <b>[Y]</b> .

Step	Display	Procedure/Comment
	Enter Treatment Number: 1	Press [ENTER] to accept current value.
	Enter No. Of Elements: 0	Type 8 and press [ENTER].
	To End Input Enter 'E'	Press [ENTER] to proceed.
	Enter X(1,1):	Type 2 and press [ENTER].
	Enter X(1,2):	Type 6 and press [ENTER].
	Enter X(1,3):	Type 4 and press [ENTER].
	Enter X(1,4):	Type 13 and press [ENTER].
	Enter X(1,5):	Type 5 and press [ENTER].
	Enter X(1,6):	Type 8 and press [ENTER].
	Enter X(1,7):	Type 4 and press [ENTER].
	Enter X(1,8):	Type 6 and press [ENTER].
	Edit?	Press [N].
	Record Data?	Press [N].
	More Treatments?	Press [Y].
	Enter Treatment Number: 2	Press [ENTER] to accept current value.
	Enter No. Of Elements: 8	Type 10 and press [ENTER].
	To End Input Enter 'E'	Press [ENTER] to proceed.
	Enter X(2,1): 0	Type 6 and press [ENTER].
	Enter X(2,2): 0	Type 4 and press [ENTER].

Step	Display	Procedure/Comment
23.	Enter X(2,3): 0	Type 4 and press [ENTER].
24.	Enter X(2,4): 0	Type 1 and press [ENTER].
25.	Enter X(2,5): 0	Type 8 and press [ENTER].
26.	Enter X(2,6): 0	Type 2 and press [ENTER].
27.	Enter X(2,7): 0	Type 12 and press [ENTER].
28.	Enter X(2,8): 0	Type 1 and press [ENTER].
29.	Enter X(2,9): 0	Type 5 and press [ENTER].
30.	Enter X(2,10): 0	Type 2 and press [ENTER].
31.	Edit?	Press [N].
32.	Record Data?	Press [N].
33.	More Treatments?	Press [Y].
34.	Enter Treatment Number: 3	Press [ENTER] to accept current value.
35.	Enter No. Of Elements: 10	Type 9 and press [CTL] [↓] to clear the rest of the field. Then press [ENTER].
36.	To End Input Enter 'E'	Press [ENTER] to proceed.
37.	Enter X(3,1): 0	Type 2 and press [ENTER].
38.	Enter X(3,2): 0	Type 1 and press [ENTER].
39.	Enter X(3,3): 0	Type 3 and press [ENTER].

Step	Display	Procedure/Comment
•	Enter X(3,4): 0	Type <b>3</b> and press <b>[ENTER]</b> .
•	Enter X(3,5): 0	Type <b>1</b> and press <b>[ENTER]</b> .
•	Enter X(3,6): 0	Type <b>7</b> and press <b>[ENTER]</b> .
•	Enter X(3,7): 0	Type <b>1</b> and press <b>[ENTER]</b> .
•	Enter X(3,8): 0	Type <b>4</b> and press <b>[ENTER]</b> .
•	Enter X(3,9): 0	Type <b>2</b> and press <b>[ENTER]</b> .
•	Edit?	Press <b>[N]</b> .
•	Record Data?	Press <b>[N]</b> .
•	More Treatments?	Press <b>[N]</b> .
•	SS(TOTAL)= 264 df(TOTAL)= 26  SST= 47.5 dfT= 2 MST= 23.75  SSE= 216.5 dfE= 24 MSE= 9.020833333  F= 2.632794457	Displays all requested values. Press <b>[ENTER]</b> after each output to proceed.
•	Exit Program?	Press <b>[Y]</b> .

### Summary

You can now enter the value for the F statistic into the F-distribution program to compare the means of the sample data for the three drugs.

## The Two-Way Analysis of Variance (AOV2) Program

The Two-Way Analysis of Variance program is used to determine if either of two factors or an interaction between the factors has a significant effect on the response.

### Two-Factor Situations

When two factors are involved, the test procedures must be repeated so that some estimate of random error can be made. To repeat (or replicate) the test procedure, you must prepare two or more independent samples. Then, you should treat these samples similarly.

### Two-Way ANOVA Notations

Notation in the Two-Way ANOVA program is the same as the notation in the One-Way ANOVA program except a subscript  $k$  is added to represent the repetition. For example, the notation  $x_{423}$  represents the third data value in the second column of the fourth treatment.

The notation  $T_{.j.}$  indicates a column total for all row and replicate numbers. The table below shows a standard two-factor experiment repeated twice. The experiment consists of mixing film ingredients for a certain time at a certain temperature. The results attained in the experiment are a percentage change of processing time from the normal processing time.

Time	Temperature		$T_{i..}$
	50	70	
1 hour	10 30 $T_{1j.}$ 40	-20 -40 -60	-20
2 hours	0 40 $T_{2j.}$ 40	50 40 90	130
$T_{.j.}$	80	30	$110 = T_{...}$



**Two-Way Analysis of Variance** The total sum of squares ( $SS\sum$ ) is found by

$$SS\sum = \sum_{k=1}^K \sum_{j=1}^c \sum_{i=1}^r x_{kij}^2 - T_{...}^2/N$$

where  $N = \text{rows} \times \text{columns} \times \text{replicates} = \text{total observations}$ .  
The sum of squares for rows is calculated by the following formula:

$$SSR = \frac{1}{ck} \sum_{i=1}^r T_{i..}^2 - T_{...}^2/N$$

where  $ck = \text{columns} \times \text{replicates}$ .

Similarly, the sum of squares for the columns is

$$SSC = \frac{1}{rk} \sum_{i=1}^c T_{.i.}^2 - T_{...}^2/N$$

where  $rk = \text{rows} \times \text{replicates}$ .

Finally, the sums of squares for interaction is found by the formula:

$$SSI = \frac{1}{k} \sum_{j=1}^c \sum_{i=1}^r T_{ij.}^2 - SSR - SSC - T_{...}^2/N$$

The sum of squares for error is derived by subtracting the above sums of squares from the total sum of squares.

$$SSE = SS\sum - SSR - SSC - SSI$$

## Two-Way Analysis of Variance (Continued)

The degrees of freedom (df) for each respective formula above are

$$df \Sigma = N - 1$$

$$MSR = SSR/df R$$

$$df R = r - 1$$

$$MSC = SSC/df C$$

$$df C = c - 1$$

$$MSI = SSI/df I$$

$$df I = (r - 1)(c - 1)$$

$$MSE = SSE/df E$$

$$df E = df \Sigma - df R - df C - df I$$

With the data previously provided in this section, the following ANOVA table may be constructed.

Source	Sum of Squares	df	Mean Square	F Ratio
Time	2,812.5	1	2,812.5	9
Temperature	312.5	1	312.5	1
Interaction	2,812.5	1	2,812.5	9
Error	1,250.0	4	312.5	
Total	7,187.5	7		

As in the One-Way ANOVA table, the mean square terms are the sum of squares divided by their respective degrees of freedom. The F ratios are computed by dividing each source mean square by the error mean square, as specified by the following formulas.

$$F_r = \frac{MSR}{MSE}$$

$$F_c = \frac{MSC}{MSE}$$

$$F_i = \frac{MSI}{MSE}$$

## Using the Two-Way ANOVA Program

To execute the Two-Way ANOVA program, type RUN "AOV2" and press [ENTER].

Step	Display	Procedure/Comment
	TWO WAY ANOVA	Wait while the program name is displayed, or press [ENTER] to continue.
	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
	Enter Device Code:	Type output device code and press [ENTER].
	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
	Enter No. Of Rows:	Type the number of rows in your sample and press [ENTER].
	Enter No. Of Columns:	Type the number of columns in your sample and press [ENTER].
	Enter No. Of Replicates:	Press [ENTER] to accept current value, or type the number of replicates in your sample and press [ENTER]. a. If you are entering data from keyboard, go to step 9. b. If you are entering data from peripheral device, go to step 8.
	Enter Device.Filename:	Type the device and filename on which data is stored and press [ENTER]. Go to step 10.
	Enter (##,##,##):	Press [ENTER] to accept current value, or type the value for $X_{ijk}$ and press [ENTER]. Repeat step until all values are entered.
	Edit?	a. To edit data, press [Y]. b. To accept data, press [N]. If accepting data entered by keyboard, go to step 18. If accepting data entered by peripheral device, go to step 20.

## Using the Two-Way ANOVA Program (Continued)

Step	Display	Procedure/Comment
11.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 13.
12.	Enter (##,##,##): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat this step until all values are edited. If editing data entered by keyboard, go to step 18. If editing data entered by peripheral device, go to step 20.
13.	Enter Row Number:	Type the number of the row you want to edit and press <b>[ENTER]</b> .
14.	Enter Column Number:	Type the number of the column you want to edit and press <b>[ENTER]</b> .
15.	Enter Replicate Number:	Type the number of the replicate you want to edit and press <b>[ENTER]</b> .
16.	Enter (##,##,##): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
17.	Edit More Elements?	a. To edit more values, press <b>[Y]</b> . Go to step 13. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 18. If accepting data entered by peripheral device, go to step 20.
18.	Record Data?	a. To record data, press <b>[Y]</b> . Go to step 19. b. To select next option, press <b>[N]</b> . Go to step 20.
19.	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> .
20.	Display Anova Table?	a. To display ANOVA table, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 26.

Step	Display	Procedure/Comment
11	SS(TOTAL)= df(TOTAL)= SSR= dfR= MSR= SSC= dfC= MSC= SSI= dfI= MSI= SSE= dfE=	Displays ANOVA table, one value at a time. Press <b>[ENTER]</b> after each output to proceed.
12	MSE=	Displays mean squares error. Press <b>[ENTER]</b> to proceed. a. If sample contains only one row or if SSI and SSE have been pooled, go to step 24. b. If sample contains more than one row or if SSI and SSE have not been pooled, go to step 23.
13	Fi=	Displays F ratio for interaction. Press <b>[ENTER]</b> to proceed.
14	Fr=	Displays F ratio for rows. Press <b>[ENTER]</b> to proceed.
15	Fc=	Displays F ratio for columns. Press <b>[ENTER]</b> to proceed. a. If interaction terms have been pooled, go to step 27. b. If interaction terms have not been pooled, go to step 26.
16	Pool SSI And SSE?	a. To pool interaction terms with error terms, press <b>[Y]</b> . Go to step 20. b. To select next option, press <b>[N]</b> .
17	Exit Program?	a. To exit program, press <b>[Y]</b> . b. To continue program, press <b>[N]</b> . Go to step 4.

## Application: Considering Two Factors

In this example, you compare the means of two populations when two factors are considered. To compare the means, you must first analyze the variances for both populations.

### Example

In manufacturing a photographic film emulsion, it is customary to mix the ingredients and allow the mixture to stand for a certain time at a certain temperature. You want to know if there is a significant interaction between time and temperature for a particular set of ingredients. Because the problem involves two factors (time and temperature) plus a possible interaction, the experiment is replicated to obtain a separate estimate of error. The results are expressed as a percentage change from the normal processing time of the film, as shown below.

Stand Time	Temperature		$T_{i..}$
	60	70	
1 hour	15	-15	$\overline{-5}$
	25	-30	
	$T_{1j}$ 40	-45	
2 hours	5	55	$\overline{140}$
	45	45	
	$T_{2j}$ 50	90	
$T_{.j.}$	90	45	$135 = T_{...}$

Test the following three hypotheses at the 95% confidence level:

1. The changes in stand time produce no effect.
2. The changes in temperature produce no effect.
3. There is no interaction of time and temperature.

To execute the Two-Way ANOVA program, type **RUN "AOV2"** and press [ENTER].

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Step	Display	Procedure/Comment
1	TWO WAY ANOVA	Wait while the program name is displayed, or press [ENTER] to continue.
2	Use Printer?	Press [N].
	Enter Data By Keyboard?	Press [Y].
4	Enter No. Of Rows: 0	Type 2 and press [ENTER].
	Enter No. Of Columns: 0	Type 2 and press [ENTER].
4	Enter No. Of Replicates: 0	Type 2 and press [ENTER].
	Enter (1,1,1): 0	Type 15 and press [ENTER].
•	Enter (1,1,2): 0	Type 25 and press [ENTER].
•	Enter (1,2,1): 0	Type - 15 and press [ENTER].
	Enter (1,2,2): 0	Type - 30 and press [ENTER].
1	Enter (2,1,1): 0	Type 5 and press [ENTER].
2	Enter (2,1,2): 0	Type 45 and press [ENTER].
3	Enter (2,2,1): 0	Type 55 and press [ENTER].
4	Enter (2,2,2): 0	Type 45 and press [ENTER].
5	Edit?	Press [N].
6	Record Data?	Press [N].

Step	Display	Procedure/Comment
17.	Display Anova Table?	Press [Y].
18.	SS(TOTAL)= 6446.875 df(TOTAL)= 7  SSR= 3003.125 dfR= 1 MSR= 3003.125  SSC= 153.125 dfC= 1 MSC= 153.125  SSI= 2278.125 dfI= 1 MSI= 2278.125  SSE= 1012.5 dfE= 4 MSE= 253.125  Fi= 9 Fr= 11.86419753 Fc= .6049382716	Displays ANOVA table, one value at a time Press [ENTER] after each output to proceed
19.	Pool SSI And SSE?	Press [Y].
20.	Display Anova Table?	Press [Y].



Step	Display	Procedure/Comment
	SS(TOTAL)= 6446.875 df(TOTAL)= 7  SSR= 3003.125 dfR= 1 MSR= 3003.125  SSC= 153.125 dfC= 1 MSC= 153.125  SSE= 3290.625 dfE= 5 MSE= 658.125  Fr= 4.563152896 Fc= .232668566	Displays ANOVA table, one value at a time. Press <b>[ENTER]</b> after each output to proceed.
--	Exit Program?	Press <b>[Y]</b> .

## Chapter 4: Parametric Test Programs

---

This chapter describes the Parametric Test Programs which compute the  $t$  statistic and the corresponding number of degrees of freedom for sample data from different populations. With this information, you can compare the means of different populations. This information is also needed to use the Student's  $t$ -Distribution program.

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## The t-Test: Paired Data (TTESTP) Program

---

This program is designed to test the hypothesis that the average difference between sample pairs is equal to zero.

---

### Method Used in the Program

The t-statistic is evaluated as:

$$t = \bar{\Delta} \sqrt{n} / s_{\bar{\Delta}} \text{ with } n - 1 \text{ degrees of freedom}$$

where  $\bar{\Delta}$  = mean of the differences between the paired values

$n$  = sample size

$s_{\bar{\Delta}}$  = standard deviation of the differences between the paired values using  $n - 1$  degrees of freedom

The equation for computing  $\bar{\Delta}$  is

$$\bar{\Delta} = \frac{\sum_{i=1}^n (x_i - y_i)}{n}$$

The equation for computing  $s_{\bar{\Delta}}$  is

$$s_{\bar{\Delta}} = \sqrt{\frac{n \sum_{i=1}^n (x_i - y_i)^2 - (\sum_{i=1}^n (x_i - y_i))^2}{n - 1}}$$

## Using the t-Test: Paired Data Program

To execute the t-Test Paired Data program, type RUN "TTESTP" and press [ENTER].

Step	Display	Procedure/Comment
	t-TEST:PAIRED	Wait while the program name is displayed, or press [ENTER] to continue.
	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
	Enter Device Code:	Type output device code and press [ENTER].
	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
	Enter No. Of Elements:	Type the maximum number of elements in your sample and press [ENTER]. a. If you enter data from keyboard, go to step 7. b. If you enter data from a peripheral device, go to step 6.
	Enter Device.Filename:	Type the device number and filename on which data is stored and press [ENTER]. Go to step 11.
	To End Input Enter 'E'	Press [ENTER] to proceed.
	Enter X(###):	Press [ENTER] to accept current value, or type the value for X and press [ENTER].
	Enter Y(###):	Press [ENTER] to accept current value, or type the value for the corresponding Y element and press [ENTER]. Repeat steps 8 and 9 until all X and Y values are entered. Go to step 10.

## Using the t-Test: Paired Data Program (Continued)

Step	Display	Procedure/Comment
10.		<ol style="list-style-type: none"><li>If you entered the maximum number of elements (specified in step 5), go to step 11.</li><li>If you entered less than the maximum number, type <b>E</b> and press <b>[ENTER]</b>. Go to step 19.</li></ol>
11.	Edit?	<ol style="list-style-type: none"><li>To edit data, press <b>[Y]</b>.</li><li>To accept data, press <b>[N]</b>. If accepting data entered by keyboard, go to step 19. If accepting data entered by peripheral device, go to step 22.</li></ol>
12.	Edit All Data?	<ol style="list-style-type: none"><li>To edit all data, press <b>[Y]</b>.</li><li>To edit single elements, press <b>[N]</b>. Go to step 15.</li></ol>
13.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> .
14.	Enter Y(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> . Repeat steps 13 and 14 until all values are edited. If editing data entered by keyboard, go to step 19. If editing data entered by peripheral device, go to step 22.
15.	Enter Element #:	Type the number of the element you want to edit and press <b>[ENTER]</b> .
16.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> .
17.	Enter Y(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
18.	Edit More Elements?	a. To edit more elements, press [Y]. Go to step 15. b. To accept data, press [N]. If accepting data entered by keyboard, go to step 19. If accepting data entered by peripheral device, go to step 22.
19.	Record Data?	a. To record data, press [Y]. b. To select next option, press [N]. Go to step 21.
20.	Enter Device.Filename:	Type the device number and filename on which to record data and press [ENTER]. Go to step 23.
21.		a. If you entered the maximum number of elements, go to step 22. b. If you entered less than the maximum number of elements, go to step 23.
22.	Input Additional Data?	a. To input additional data, press [Y]. Go to step 7. b. To select next option, press [N].
23.	Standard Dev=	Displays standard deviation. Press [ENTER] to proceed.
24.	Delta X=	Displays mean of differences between pairs. Press [ENTER] to proceed.
25.	t=	Displays t-statistic. Press [ENTER] to proceed.
26.	df=	Displays degrees of freedom. Press [ENTER] to proceed.
27.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

## Application: Comparing the Means of Two Populations

The t-Test: Paired program can be used to compare the means of two corresponding sets of sample data from two populations that are normally distributed.

### Example

In an experiment to compare two different diets for pigs, a farmer randomly selects a pair of pigs from each of ten litters. One pig from each pair is placed on diet A for a fixed period of time. The other pig from each pair is placed on diet B during the same period. At the end of the experiment, the farmer weighs each pig to see how much it has gained. The results are tabulated below.

	1	2	3	4	5	6	7	8	9	10
Diet A	21.5	18.0	14.7	19.3	21.7	22.9	22.3	19.1	13.3	19.8
Diet B	14.7	16.1	15.2	14.6	17.5	15.6	20.8	20.3	12.0	20.9

Evaluate the t-statistic for paired observations at a 95% confidence level to test the null hypothesis

$H_0$ : The diets cause the same average weight gain ( $\mu_a = \mu_b$ )

against the alternate hypothesis

$H_1$ : The diets cause different average weight gains ( $\mu_a \neq \mu_b$ )

To execute the t-test: Paired program, type **RUN "TTESTP"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	t-TEST: PAIRED	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter Data By Keyboard?	Press <b>[Y]</b> .
4.	Enter No. Of Elements: 0	Type <b>10</b> and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
7.	To End Input Enter 'E'	Press [ENTER] to proceed.
8.	Enter X(1): 0	Type 21.5 and press [ENTER].
9.	Enter Y(1): 0	Type 14.7 and press [ENTER].
10.	Enter X(2): 0	Type 18 and press [ENTER].
11.	Enter Y(2): 0	Type 16.1 and press [ENTER].
12.	Enter X(3): 0	Type 14.7 and press [ENTER].
13.	Enter Y(3): 0	Type 15.2 and press [ENTER].
14.	Enter X(4): 0	Type 19.3 and press [ENTER].
15.	Enter Y(4): 0	Type 14.6 and press [ENTER].
16.	Enter X(5): 0	Type 21.7 and press [ENTER].
17.	Enter Y(5): 0	Type 17.5 and press [ENTER].
18.	Enter X(6): 0	Type 22.9 and press [ENTER].
19.	Enter Y(6): 0	Type 15.6 and press [ENTER].
20.	Enter X(7): 0	Type 22.3 and press [ENTER].
21.	ENTER Y(7): 0	Type 20.8 and press [ENTER].
22.	ENTER X(8): 0	Type 19.1 and press [ENTER].
23.	ENTER Y(8): 0	Type 20.3 and press [ENTER].
24.	ENTER X(9): 0	Type 13.3 and press [ENTER].
25.	ENTER Y(9): 0	Type 12 and press [ENTER].



## Application: Comparing the Means of Two Populations (Continued)

Step	Display	Procedure/Comment
24.	ENTER X(10): 0	Type <b>19.8</b> and press <b>[ENTER]</b> .
25.	ENTER Y(10): 0	Type <b>20.9</b> and press <b>[ENTER]</b> .
26.	Edit?	a. If you entered the data correctly, press <b>[N]</b> . b. If you would like to recheck your data, press <b>[Y]</b> .
27.	Record Data?	Press <b>[N]</b> .
28.	Input Additional Data?	Press <b>[N]</b> .
29.	Standard Dev= 3.121769441 Delta X= 2.49 t= 2.522310351 df= 9	Displays all requested values. Proceed after each output by pressing <b>[ENTER]</b> .
30.	Exit Program?	Press <b>[Y]</b> .

## The t-Test: Unpaired Data (TTESTU) Program

---

This program is designed to test the hypothesis that two normally distributed populations with the same unknown variance have a difference in mean of  $\Delta$ . If samples of two normal populations have the same variance and consist of unpaired elements, you can use this program to compute the t-statistic and the number of degrees of freedom.

---

### Comparing the Means of Two Populations

To compute the t-statistic and the number of degrees of freedom with this program, you must be able to assume that two normal populations have the same unknown variance, and your data values must not be paired.

### Method Used in the Program

The t-statistic for two normally distributed populations with unpaired data values is determined from the equation

$$t = \frac{\bar{x} - \bar{y} - \Delta}{\left( \frac{1}{n_x} + \frac{1}{n_y} \right)^{1/2} \left( \frac{\sum_{i=1}^{n_x} x_i^2 - n_x \bar{x}^2 + \sum_{j=1}^{n_y} y_j^2 - n_y \bar{y}^2}{n_x + n_y - 2} \right)^{1/2}}$$

where

$\Delta$  = the difference between the means of the X and Y populations.

$n_x$  = the number of X observations.

$n_y$  = the number of Y observations.

This formula uses  $n_x + n_y - 2$  degrees of freedom.

## Using the t-Test: Unpaired Data Program

To execute the t-Test: Unpaired program, type RUN "TTESTU" and press [ENTER].

Step	Display	Procedure/Comment
1.	t-TEST:UNPAIRED	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use a printer, press [Y]. b. If you do not want to use a printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from a peripheral device, press [N].
5.	Is Data Grouped?	a. To select grouped data, press [Y]. b. To select ungrouped data, press [N].
6.	Input X Values...	Displays prompt. Press [ENTER] to proceed.
7.	Enter No. Of Elements:	Press [ENTER] to accept current value, or type the maximum number of elements in sample 1 and press [ENTER]. a. If entering data by keyboard, go to step 9. b. If entering data by peripheral device, go to step 8.
8.	Enter Device.Filename:	Type the device number and filename on which data is stored and press [ENTER]. Go to step 13.
9.	To End Input Enter 'E'	Press [ENTER] to proceed.

Step	Display	Procedure/Comment
10.	Enter X(###):	Press <b>[ENTER]</b> to accept current value, or type value for $X_i$ and press <b>[ENTER]</b> . a. If you enter the maximum number of elements and they are ungrouped, repeat this step until all values are entered. Go to step 12. b. If the elements are grouped, go to step 11. c. If you do not want to enter the maximum number of elements, type <b>E</b> and press <b>[ENTER]</b> . Go to step 12.
11.	Enter f(###):	Press <b>[ENTER]</b> to accept current value, or type the frequency for cell $i$ and press <b>[ENTER]</b> . Repeat steps 10 and 11 until all values are entered.
12.	Edit?	a. To edit data, press <b>[Y]</b> . b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 20. If accepting data entered by peripheral device, go to step 22.
13.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 16.
14.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> . a. If data is grouped, go to step 15. b. If data is ungrouped and entered by keyboard, repeat this step until all values are edited. Go to step 20. c. If data is ungrouped and entered by peripheral device, repeat this step until all values are edited. Go to step 22.

## Using the t-Test: Unpaired Data Program (Continued)

Step	Display	Procedure/Comment
15.	Enter f(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat steps 14 and 15 until all values are edited. If editing data entered by keyboard, go to step 20. If editing data entered by peripheral device, go to step 22.
16.	Enter Element #:	Type the number of the element you want to edit and press <b>[ENTER]</b> .
17.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . a. If data is ungrouped, go to step 19. b. If data is grouped, go to step 18.
18.	Enter f(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
19.	Edit More Elements?	a. To edit more elements, press <b>[Y]</b> . Go to step 16. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 20. If accepting data entered by peripheral device, go to step 22.
20.	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . If you entered the maximum number of elements, go to step 22. If you entered less than the maximum number, go to step 23.
21.	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> .
22.	Input Additional Data?	a. To input additional data, press <b>[Y]</b> . Go to step 9. b. To select next option, press <b>[N]</b> .
23.	Input Y Values...	Press <b>[ENTER]</b> to proceed.

Step	Display	Procedure/Comment
24.	Enter No. Of Elements:	Press <b>[ENTER]</b> to accept current value, or type the maximum number of elements in sample 2 and press <b>[ENTER]</b> . a. If entering data by keyboard, go to step 26. b. If entering data by peripheral device, go to step 25.
25.	Enter Device.Filename:	Type the device number and filename on which data is stored and press <b>[ENTER]</b> . Go to step 29.
26.	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
27.	Enter Y(###):	Press <b>[ENTER]</b> to accept current value, or type the value for $Y_i$ and press <b>[ENTER]</b> . a. If you enter the maximum number of elements and they are ungrouped, repeat this step until all values are entered. Go to step 29. b. If the elements are grouped, go to step 28. c. If you do not want enter the maximum number of elements, type <b>E</b> and press <b>[ENTER]</b> . Go to step 29.
28.	Enter f(###):	Press <b>[ENTER]</b> to accept current value, or type the frequency for cell $i$ and press <b>[ENTER]</b> . Repeat steps 27 and 28 until all values are entered. Go to step 29.
29.	Edit?	a. To edit data, press <b>[Y]</b> . b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 37. If accepting data entered by peripheral device, go to step 39.

## Using the t-Test: Unpaired Data Program (Continued)

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Step	Display	Procedure/Comment
30.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 33.
31.	Enter Y(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . a. If data is grouped, go to step 32. b. If data is ungrouped and entered by keyboard, repeat this step until all values are edited. Go to step 37. c. If data is ungrouped and entered by peripheral device, repeat this step until all values are edited. Go to step 39.
32.	Enter f(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> . Repeat steps 31 and 32 until all values are edited. If editing data entered by keyboard, go to step 37. If editing data entered by peripheral device, go to step 39.
33.	Enter Element #:	Type the number of the element you want to edit and press <b>[ENTER]</b> .
34.	Enter Y(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . a. If data is grouped, go to step 35. b. If data is ungrouped and entered by keyboard, repeat this step until all values are edited. Go to step 37. c. If data is ungrouped and entered by peripheral device, repeat this step until all values are edited. Go to step 39.
35.	Enter f(###): ####	Press <b>[ENTER]</b> to accept current value, or type the correct value and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
36.	Edit More Elements?	a. To edit more elements, press <b>[Y]</b> . Go to step 33. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 37. If accepting data entered by peripheral device, go to step 39.
37.	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . If you entered the maximum number of elements, go to step 39. If you entered less than the maximum number, go to step 40.
38.	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> . Go to step 40.
39.	Input Additional Data?	a. To input additional data, press <b>[Y]</b> . Go to step 27. b. To select next option, press <b>[N]</b> . Go to step 40.
40.	Enter Hyp. Delta Mean:	Type the hypothesized mean difference and press <b>[ENTER]</b> .
41.	t=	Displays t-statistic. Press <b>[ENTER]</b> to proceed.
42.	df=	Displays degrees of freedom. Press <b>[ENTER]</b> to proceed.
43.	nx=	Displays number of X entries. Press <b>[ENTER]</b> to proceed.
44.	ny=	Displays number of Y entries. Press <b>[ENTER]</b> to proceed.
45.	Exit Program?	a. To exit program, press <b>[Y]</b> . b. To continue program, press <b>[N]</b> . Go to step 4.



## Application: Comparing Samples with Unpaired Elements

In this example, the two samples are taken from populations that are normally distributed with unknown variances that are assumed to be equal. The elements in the two samples, however, cannot be paired.

### Example

Two groups of patients at a major hospital are selected for an experiment to compare two drugs used for relief of pain. One group is given drug x, and the other group is given drug y. The resulting number of hours of relief for each patient is given below.

Drug x: 2, 6, 4, 13, 5, 8, 4, 6.

Drug y: 6, 4, 4, 1, 8, 2, 12, 1, 5, 2.

Use the Unpaired t-Test program to test the null hypothesis

$$H_0: \mu_x - \mu = 2 \text{ hours}$$

against the alternate hypothesis

$$H_1: \mu_x - \mu \neq 2 \text{ hours.}$$

To execute the Unpaired t-test program, type **RUN**  
**"TTESTU"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	t-TEST:UNPAIRED	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter Data By Keyboard?	Press <b>[Y]</b> .
4.	Is Data Grouped?	Press <b>[N]</b> .
5.	Input X Values:...	Press <b>[ENTER]</b> to proceed.
6.	Enter No. Of Elements: 0	Type <b>10</b> and press <b>[ENTER]</b> .
7.	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
8.	Enter X(1): 0	Type <b>2</b> and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
9.	Enter X(2): 0	Type <b>6</b> and press <b>[ENTER]</b> .
10.	Enter X(3): 0	Type <b>4</b> and press <b>[ENTER]</b> .
11.	Enter X(4): 0	Type <b>13</b> and press <b>[ENTER]</b> .
12.	Enter X(5): 0	Type <b>5</b> and press <b>[ENTER]</b> .
13.	Enter X(6): 0	Type <b>8</b> and press <b>[ENTER]</b> .
14.	Enter X(7): 0	Type <b>4</b> and press <b>[ENTER]</b> .
15.	Enter X(8): 0	Type <b>6</b> and press <b>[ENTER]</b> .
16.	Enter X(9): 0	Type <b>E</b> and press <b>[ENTER]</b> to end data input.
17.	Edit?	Press <b>[N]</b> .
18.	Record Data?	Press <b>[N]</b> .
19.	Input Y Values...	Press <b>[ENTER]</b> to proceed.
20.	Type No. Of Elements: 0	Type <b>12</b> and press <b>[ENTER]</b> .
21.	To End Input Enter 'E'	Press <b>[ENTER]</b> to proceed.
22.	Enter Y(1): 0	Type <b>6</b> and press <b>[ENTER]</b> .
23.	Enter Y(2): 0	Type <b>4</b> and press <b>[ENTER]</b> .
24.	Enter Y(3): 0	Type <b>4</b> and press <b>[ENTER]</b> .
25.	Enter Y(4): 0	Type <b>1</b> and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
26.	Enter Y(5): 0	Type 8 and press [ENTER].
27.	Enter Y(6): 0	Type 2 and press [ENTER].
28.	Enter Y(7): 0	Type 12 and press [ENTER].
29.	Enter Y(8): 0	Type 1 and press [ENTER].
30.	Enter Y(9): 0	Type 5 and press [ENTER].
31.	Enter Y(10): 0	Type 2 and press [ENTER].
32.	Enter Y(11): 0	Type E and press [ENTER] to end data input.
33.	Edit?	Press [N].
34.	Record Data?	Press [N].
35.	Enter Hyp. Delta Mean: 0	Type 2 and press [ENTER].
36.	t=-.3087445631 df= 16 nx= 8 ny= 10	Displays all requested values. Proceed after each output by pressing [ENTER].
37.	Exit Program?	Press [Y].

## Chapter 5: Non-Parametric Test Programs

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This chapter describes the Non-Parametric Test programs which use the chi-square test of independence and a contingency table to compute the degrees of freedom for samples involving two factors, and the Mann-Whitney Rank-Sum test to compare the means of two populations with the same unknown distribution.

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# The Contingency Table Analysis (CTBL) Program

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By analyzing a contingency table, you can determine whether the two factors are independent of each other. The Contingency Table Analysis program applies the chi-square test of independence to find the degrees of freedom and the cumulative distribution function of  $\chi^2$ .

---

## Method Used in the Program

You can use the Contingency Table Analysis program to apply the chi-square test of independence to the row and column classifications of a contingency table. This program computes the:

- Chi-square statistic, denoted by  $\chi^2$ , with  $(R-1)(C-1)$  degrees of freedom (where R is the number of rows and C is the number of columns)
- Cumulative distribution function of  $\chi^2$ ,  $P(\chi^2)$ .

You fail to reject a null hypothesis when  $P(\chi^2)$  is less than or equal to the specified confidence level.

Calculations are based upon  $\chi^2$  where

$$\chi^2 = \sum_{i=1}^R \sum_{j=1}^C (N_{ij} - E_{ij})^2 / E_{ij}$$

with  $(R-1)(C-1)$  degrees of freedom

where R = the number of rows

C = the number of columns

$N_{ij}$  = the count, or number of observations occurring in cell ij (row i, column j)

$E_{ij}$  = the maximum likelihood estimator of the number of observations that should occur in cell ij when the classifications are independent (the row and column factors are unrelated).

The probability of an event falling in both row i and column j is expressed as

$$E_{ij} = (N_{i.} \cdot N_{.j}) / n$$

where:  $N_{i.}$  = the total number of counts in the ith row

$N_{.j}$  = the total number of counts in the jth column

n = the total number of observations,  $N_{..}$

## Reference

*Practical Non-Parametric Statistics*, Conover, Wiley, 1971, pp. 154-155.

## Using the Contingency Table Analysis Program

To execute the Contingency Table Analysis program, type RUN "CTBL" and press [ENTER].

Step	Display	Procedure/Comment
1.	CONTINGENCY TABLE	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
5.	Enter No. Of Rows:	Press [ENTER] to accept current value, or type the number of rows in the sample and press [ENTER].
6.	Enter No. Of Columns:	Press [ENTER] to accept current value, or type the number of columns in the sample and press [ENTER]. a. If entering data from keyboard, go to step 9. b. If entering data from a peripheral device, go to step 8.
8.	Enter Device.Filename:	Type the device number and filename on which data is stored and press [ENTER]. Go to step 10.
9.	Enter X(##,##):	Press [ENTER] to accept current value, or type value for $X_{ij}$ and press [ENTER]. Repeat this step until all values are entered.
10.	Edit?	a. To edit data, press [Y]. b. To accept data, press [N]. If accepting data entered by keyboard, go to step 17. If accepting data entered by peripheral device, go to step 19.

## Using the Contingency Table Analysis Program (Continued)

Step	Display	Procedure/Comment
11.	Edit All Data?	a. To edit all data, press [Y]. b. To edit single elements, press [N]. Go to step 13.
12.	Enter X(##,##): ####	Press [ENTER] to accept current value, or type correct value and press [ENTER]. Repeat this step until all values are edited. If editing data entered by keyboard, go to step 17. If editing data entered by peripheral device, go to step 19.
13.	Enter Row Number:	Type the number of the row you want to edit and press [ENTER].
14.	Enter Column Number:	Type the number of the column you want to edit and press [ENTER].
15.	Enter X(##,##): ####	Press [ENTER] to accept current value, or type correct value and press [ENTER].
16.	Edit More Elements?	a. To edit more values, press [Y]. Go to step 13. b. To accept data, press [N]. If accepting data entered by keyboard, go to step 17. If accepting data entered by peripheral device, go to step 19.
17.	Record Data?	a. To record data, press [Y]. b. To select next option, press [N]. Go to step 19.
18.	Enter Device.Filename:	Type the device number and filename on which to record data and press [ENTER].
19.	Chi-Square=	Displays the chi-square statistic. Press [ENTER] to proceed.

---

---

Step	Display	Procedure/Comment
20.	df=	Displays degrees of freedom. Press <b>[ENTER]</b> to proceed.
21.	P(##)=	Displays area of the curve to the to the left of the chi-square. Press <b>[ENTER]</b> to proceed.
22.	Exit Program?	a. To exit program, press <b>[Y]</b> . b. To continue program, press <b>[N]</b> . Go to step 4.

---



## Application: Influence of Property Ownership on Voter Opinion

The Contingency Table program enables you to test the hypothesis that two classifications are independent of one another. In this example, you use the program to determine if a voter's opinion on an issue is influenced by the fact that he is a property owner.

### Example

Two hundred voters in a local bond election are randomly selected and asked their opinion on an issue. The voters are then classified according to their answers to the question and whether or not they are property owners as illustrated below.

	For	Against	Undecided	$N_{i.}$
Property Owner	45	39	21	105
Non-Property Owner	47	26	22	95
$N_{.j}$	92	65	43	200

Test the null hypothesis that a voter's opinion of the bond issue is independent of his status as a property owner. Use a 90% confidence level.

To execute the Contingency Table program, type **RUN "CTBL"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	CONTINGENCY TABLE	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter Data By Keyboard?	Press <b>[Y]</b> .
4.	Enter No. Of Rows: 0	Type <b>2</b> and press <b>[ENTER]</b> .
5.	Enter No. Of Columns: 0	Type <b>3</b> and press <b>[ENTER]</b> .
6.	Enter $X(1,1)$ : 0	Type <b>45</b> and press <b>[ENTER]</b> .
7.	Enter $X(1,2)$ : 0	Type <b>39</b> and press <b>[ENTER]</b> .
8.	Enter $X(1,3)$ : 0	Type <b>21</b> and press <b>[ENTER]</b> .

Step	Display	Procedure/Comment
9.	Enter X(2,1): 0	Type <b>47</b> and press <b>[ENTER]</b> .
10.	Enter X(2,2): 0	Type <b>26</b> and press <b>[ENTER]</b> .
11.	Enter X(2,3): 0	Type <b>22</b> and press <b>[ENTER]</b> .
12.	Edit?	Press <b>[N]</b> .
13.	Record Data?	Press <b>[N]</b> .
14.	Chi-Square= 2.172164486	Displays the chi-square statistic. Press <b>[ENTER]</b> to proceed.
15.	df= 2	Displays degrees of freedom. Press <b>[ENTER]</b> to proceed.
16.	P(2.172164486)= .6624637081	Displays area of the curve to the left of the chi-square. Press <b>[ENTER]</b> to proceed.
17.	Exit Program?	Press <b>[Y]</b> .

### Summary

To evaluate this statistic, you would normally determine an acceptance region for the computed statistic.  $\chi^2$  depends upon the number of degrees of freedom of your test statistic and the confidence interval at which you are testing. At the 90% confidence level, this interval would be (0,4.6)\* for a  $\chi^2$  with 2 degrees of freedom. Because the value calculated for  $\chi^2$  falls within this interval, you fail to reject the null hypothesis. Therefore, your conclusion is: there is no evidence that status as a property owner influences a voter's opinion on an issue.

The program also computes  $P(\chi^2)$ . This chi value ranges over the interval (0,1). When testing at the 90% confidence level,  $P(\chi^2) = 0.90$ . Thus, this test fails to reject  $H_0$  because  $P(\chi^2) \leq 0.90$ .

\*This value may be verified with the Chi-Square program.

# The Mann-Whitney Rank-Sum Test (RSUM) Program

---

If two populations are assumed to have the same distribution, you can use the Mann-Whitney Rank-Sum Test program to compare the means of the two populations even if the form of the distribution is unknown.

---

## Comparing Means

This program compares the means of two populations where:

- ▶  $X$  is a random sample of size  $m$ .
- ▶  $Y$  is a random sample of size  $n$ .
- ▶ The two distributions of the cumulative density functions  $f(x)$  and  $g(x)$  take the same form.

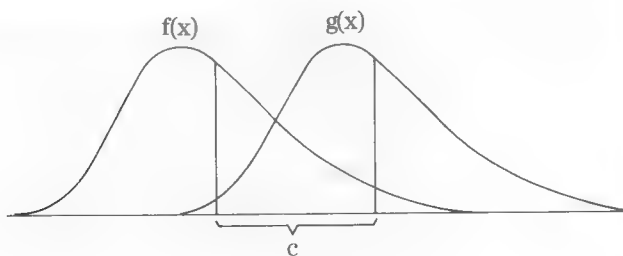
You use this program to test the null hypothesis

$H_0$ : the means of the populations are equal;  $f(x) = g(x)$

against the alternate hypothesis

$H_1$ : the means of the populations differ by an unknown constant  $c$ , such that  $f(x) = g(x + c)$ .

The following graph illustrates  $H_1$ , as  $g(x)$  has shifted to the right by an amount  $c$ .



---

**Method Used in  
the Program**

The program uses the following method to determine whether or not the means of the two populations are equal.

1. The program ranks the values in your two samples from smallest to largest. For example, if  $X = (2, 5, 6)$  and  $Y = (3, 6, 7)$ , the combined ordered set would be

$$(x_1, y_1, x_2, x_3, y_2, y_3)$$

The assigned ranks would be (1, 2, 3, 4.5, 4.5, 6) respectively. (This sample size is used for convenience; a larger sample would provide more accurate results.)

2. When  $H_1$  is true for a positive  $c$ , the  $y$  values tend to be larger than the  $x$  values. If  $R(y_i)$  is the rank of  $y_i$  and  $T_y$  is the sum of the  $y$  ranks, then the program uses the equation

$$T_y = \sum_{i=1}^n R(y_i)$$

If you substitute the  $X$  and  $Y$  values from the previous page,  $T_y = 2 + 4.5 + 6 = 12.5$ .

3. If  $H_0$  is true and  $m$  and  $n$  are both greater than 9, then  $T_y$  possesses an approximately normal distribution. Consequently, the program can use this formula to find the Mann-Whitney statistic for  $y$ :

$$w_y = mn - T_y + n(n+1)/2$$

and to find that the  $y$  statistic is nearly normally distributed.

---

### Method Used in the Program (Continued)

4. The program calculates the mean and variance of  $w_y$  with the equations:

$$w = mn/2 \text{ and } s_w^2 = mn(m+n+1)/12$$

5. The program then converts the standard deviation to standard form, yielding

$$z_y = (w_y - \bar{w})/s_w$$

where  $z$  is the standard deviate for that particular Mann-Whitney statistic.

You can now use this data in the Normal Distribution program to perform a lower-tailed test that evaluates your hypothesis.

**Note:** If the difference between the means ( $c$ ) might be a negative number, you should use  $T_x$  as the basis of your test. The program uses the same calculations except that an upper-tailed test is used in the final evaluation.

### Reference

*Practical Non-Parametric Statistics*, Conover, Wiley, 1971 pp. 224-229.

## Using the Mann-Whitney Rank-Sum Test Program

To execute the Mann-Whitney Rank-Sum Test program, type RUN "RSUM" and press [ENTER].

Step	Display	Procedure/Comment
1.	RANK SUM TEST	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter Data By Keyboard?	a. To enter data from keyboard, press [Y]. b. To enter data from peripheral device, press [N].
	Enter Size For Sample 1:	Press [ENTER] to accept current value, or type the number of elements in sample 1 and press [ENTER].
	Enter Size For Sample 2:	Press [ENTER] to accept current value, or type the number of elements in sample 2 and press [ENTER].
	Input Sample One...	Press [ENTER] to proceed. a. If entering data from keyboard, go to step 9. b. If entering data from peripheral device, go to step 8.
	Enter Device.Filename:	Type the device number and filename on which data is stored and press [ENTER]. Go to step 10.
	Enter X(###):	Press [ENTER] to accept current value, or type the value for $X_i$ and press [ENTER]. Repeat this step until all the sample 1 values are entered.
	Edit?	a. To edit data, press [Y]. b. To accept data, press [N]. If accepting data entered by keyboard, go to step 16. If accepting data entered by peripheral device, go to step 18.

## Using the Mann-Whitney Rank-Sum Test Program (Continued)

Step	Display	Procedure/Comment
11.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 13.
12.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat this step until all values are edited. If editing data entered by keyboard, go to step 16. If editing data entered by peripheral device, go to step 18.
13.	Enter Element #:	Type the number of the element you want to edit and press <b>[ENTER]</b> .
14.	Enter X(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
15.	Edit More Elements?	a. To edit more elements, press <b>[Y]</b> . Go to step 14. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 16. If accepting data entered by peripheral device, go to step 18.
16.	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 18.
17.	Enter Device.Filename:	Type the device number and filename on which to record data and press <b>[ENTER]</b> .
18.	Input Sample Two...	Press <b>[ENTER]</b> to proceed. a. If entering data from keyboard, go to step 20. b. If entering data from a peripheral device, go to step 19.
19.	Enter Device.Filename:	Type the device and filename on which data is stored and press <b>[ENTER]</b> . Go to step 21.

Step	Display	Procedure/Comment
20.	Enter Y(###):	a. Press <b>[ENTER]</b> to accept current value, or type the value for Y <sub>i</sub> and press <b>[ENTER]</b> . Repeat this step until all the sample 2 values are entered. b. If you do not want to enter the total number of elements specified in step 6, type <b>E</b> and press <b>[ENTER]</b> to indicate end of data input.
21.	Edit?	a. To edit data, press <b>[Y]</b> . b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 27. If accepting data entered by peripheral device, go to step 29.
22.	Edit All Data?	a. To edit all data, press <b>[Y]</b> . b. To edit single elements, press <b>[N]</b> . Go to step 24.
23.	Enter Y(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> . Repeat this step until all values are edited. Go to step 27.
24.	Enter Element #:	Type the number of the element you want to edit and press <b>[ENTER]</b> .
25.	Enter Y(###): ####	Press <b>[ENTER]</b> to accept current value, or type correct value and press <b>[ENTER]</b> .
26.	Edit More Elements?	a. To edit more elements, press <b>[Y]</b> . Go to step 24. b. To accept data, press <b>[N]</b> . If accepting data entered by keyboard, go to step 27. If accepting data entered by peripheral device, go to step 29.
27.	Record Data?	a. To record data, press <b>[Y]</b> . b. To select next option, press <b>[N]</b> . Go to step 29.



## Using the Mann-Whitney Rank-Sum Test Program (Continued)

Step	Display	Procedure/Comment
28.	Enter Device.Filename:	Type the device number and filename on which to record data and press [ENTER].
29.	See Rank And Values?	a. To display values and their rank based on merged data, press [Y]. b. To select next option, press [N].
30.	X(#)= Rank=	Displays each value and its rank. Press [ENTER] after each output to proceed.
31.	See X-Statistics?	a. To display X-statistics, press [Y]. b. To select next option, press [N]. Go to step 33.
32.	Sum Of X-Ranks= W(X)= Z(X)=	Displays sum of X-ranks, Mann-Whitney X-statistic, and standard deviate for X-statistic, respectively. Press [ENTER] after each output to proceed.
33.	See Y-Statistics?	a. To display Y-statistics, press [Y]. b. To select next option, press [N]. Go to step 35.
34.	Sum Of Y-Ranks= W(Y)= Z(Y)=	Displays sum of Y-ranks, Mann-Whitney Y-statistic, and standard deviate for Y-statistic, respectively. Press [ENTER] after each output to proceed.
35.	Rank Mean=	Displays the rank mean. Press [ENTER] to proceed.
36.	Rank Variance=	Displays the rank variance. Press [ENTER] to proceed.
37.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

## Application: Comparing the Heights of Fathers and Sons

---

This example uses the same sample data as the example in the Means and Moments chapter. In this example, however, you are using the Mann-Whitney Rank-Sum program to compare the means of both populations.

---

### Example

In the Means and Moments application example, the distribution of the heights of a group of men had the same shape as the distribution of the heights of their sons. The only difference detected was that the second distribution appeared to be shifted slightly to the right. The data used in the example is repeated below.

Fathers: 67.2, 65.0, 68.3, 69.9, 66.3, 69.7, 69.5, 72.9, 70.2, 74.1.

Sons: 68.4, 65.3, 66.5, 69.0, 73.6, 75.9, 69.7, 69.8, 71.0, 70.8, 67.7, 74.4, 69.9, 71.5, 71.1.

Use the Mann-Whitney Rank-Sum program to test the hypothesis

$H_0$ : the means of the distribution are equal;  $f(x) = g(x)$

against the alternate hypothesis

$H_1$ : the means of the distributions differ by a positive constant  $c$ , such that  $f(x) = g(x + c)$ .

Perform this test at the 95% confidence level.

To execute the Rank Sum Test program, type **RUN "RSUM"** and press **[ENTER]**.

---

Step	Display	Procedure/Comment
1.	RANK SUM TEST	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter Data By Keyboard?	Press <b>[Y]</b> .
4.	Enter Size For Sample 1: 0	Type <b>10</b> and press <b>[ENTER]</b> .
5.	Enter Size For Sample 2: 0	Type <b>15</b> and press <b>[ENTER]</b> .

---

## Application: Comparing the Heights of Fathers and Sons (Cont.)

Step	Display	Procedure/Comment
6.	Input Sample One...	Press [ENTER] to proceed.
7.	Enter X(1): 0	Type 67.2 and press [ENTER].
8.	Enter X(2): 0	Type 65 and press [ENTER].
9.	Enter X(3): 0	Type 68.3 and press [ENTER].
10.	Enter X(4): 0	Type 69.9 and press [ENTER].
11.	Enter X(5): 0	Type 66.3 and press [ENTER].
12.	Enter X(6): 0	Type 69.7 and press [ENTER].
13.	Enter X(7): 0	Type 69.5 and press [ENTER].
14.	Enter X(8): 0	Type 72.9 and press [ENTER].
15.	Enter X(9): 0	Type 70.2 and press [ENTER].
16.	Enter X(10): 0	Type 74.1 and press [ENTER].
17.	Edit?	Press [N].
18.	Record Data?	Press [N].
19.	Input Sample Two...	Press [ENTER] to proceed.
20.	Enter Y(1): 0	Type 68.4 and press [ENTER].
21.	Enter Y(2): 0	Type 65.3 and press [ENTER].
22.	Enter Y(3): 0	Type 66.5 and press [ENTER].
23.	Enter Y(4): 0	Type 69 and press [ENTER].

---

---

Step	Display	Procedure/Comment
24.	Enter Y(5): 0	Type <b>73.6</b> and press <b>[ENTER]</b> .
25.	Enter Y(6): 0	Type <b>75.9</b> and press <b>[ENTER]</b> .
26.	Enter Y(7): 0	Type <b>69.7</b> and press <b>[ENTER]</b> .
27.	Enter Y(8): 0	Type <b>69.8</b> and press <b>[ENTER]</b> .
28.	Enter Y(9): 0	Type <b>71</b> and press <b>[ENTER]</b> .
29.	Enter Y(10): 0	Type <b>70.8</b> and press <b>[ENTER]</b> .
30.	Enter Y(11): 0	Type <b>67.7</b> and press <b>[ENTER]</b> .
31.	Enter Y(12): 0	Type <b>74.4</b> and press <b>[ENTER]</b> .
32.	Enter Y(13): 0	Type <b>69.9</b> and press <b>[ENTER]</b> .
33.	Enter Y(14): 0	Type <b>71.5</b> and press <b>[ENTER]</b> .
34.	Enter Y(15): 0	Type <b>71.1</b> and press <b>[ENTER]</b> .
35.	Edit?	Press <b>[N]</b> .
36.	Record Data?	Press <b>[N]</b> .
37.	See Rank And Values?	Press <b>[Y]</b> .

## Application: Comparing the Heights of Fathers and Sons (Cont.)

Step	Display	Procedure/Comment
38.	X(1)= 65 Rank= 1  Y(1)= 65.3 Rank= 2  X(2)= 66.3 Rank= 3  Y(2)= 66.5 Rank= 4 : : X(10)= 74.1 Rank= 23  Y(14)= 74.4 Rank= 24  Y(15)= 75.9 Rank= 25	Displays all requested values one at a time. Press [ENTER] after each output to proceed.
39.	See X-Statistics?	Press [Y].
40.	Sum Of X-Ranks= 112 W(X)= 93 Z(X)= .9984603532	Displays all requested values one at a time. Press [ENTER] after each output to proceed.
41.	See Y-Statistics?	Press [Y].
42.	Sum Of Y-Ranks= 213 W(Y)= 57 Z(Y)= -.9984603532 Rank Mean= 75 Rank Variance= 325	Displays all requested values one at a time. Press [ENTER] after each output to proceed.
43.	Exit Program?	Press [Y].

## Chapter 6: Theoretical Distributions Programs

---

This chapter describes the Theoretical Distributions programs, which enable you to reject or fail to reject your hypothesis by selecting a confidence level and then determining an acceptance region. If the data values do not fall in the acceptance region, you reject the hypothesis.

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# The Binomial Distribution (BINOM) Program

---

This program computes probabilities based on the binomial distribution.

---

## Calculating Probabilities

You can use this program to calculate the following probabilities for  $n$  trials of an experiment.

- ▶ Probability of exactly  $k$  successes =  $f(k; n, p) = f(k)$ .
- ▶ Probability of  $k$  or fewer successes =  $F(k; n, p) = P(k)$ .
- ▶ Probability of more than  $k$  successes =  $1 - F(k; n, p) = Q(k)$ .

## Method Used in the Program

The probability function of a binomial distribution is given by

$$f(k; n, p) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & k = 0, 1, 2, \dots, n; \\ 0, & \text{elsewhere} \end{cases}$$

The cumulative distribution function of a random variable with binomial distribution is simply

$$F(k; n, p) = \sum_{j=0}^k f(j; n, p)$$

The mean and standard deviation are determined by

$$\bar{x} = np$$

and

$$\sigma = np(1-p)$$

## Using the Binomial Distribution Program

To execute the Binomial Distribution program, type RUN "BINOM" and press [ENTER].

Step	Display	Procedure/Comment
1.	BINOMIAL DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter n:	Press [ENTER] to accept current value, or type number of trials and press [ENTER].
5.	Enter p:	Press [ENTER] to accept current value, or type probability of success and press [ENTER].
6.	Enter k:	Press [ENTER] to accept current value, or type number of successes and press [ENTER].
7.	Mean=	Displays the mean. Press [ENTER] to proceed.
8.	Standard Dev=	Displays the standard deviation. Press [ENTER] to proceed.
9.	$f(k)=$	Displays the probability of exactly k successes. Press [ENTER] to proceed.
10.	$P(k)=$	Displays the probability of k or fewer successes. Press [ENTER] to proceed.
11.	$Q(k)=$	Displays the probability of more than k successes. Press [ENTER] to proceed.
12.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.



## Application: Rolling Dice

---

In this example, you use the Binomial Distribution program to compute probabilities when rolling a balanced die.

### Example

Find the probability of rolling exactly 2 fours in 5 rolls of a balanced die.

To execute the Binomial Distribution program, type **RUN "BINOM"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	BINOMIAL DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter n: 0	Type <b>5</b> and press <b>[ENTER]</b> .
4.	Enter p: 0	Type <b>1/6</b> and press <b>[ENTER]</b> .
5.	Enter k: 0	Type <b>2</b> and press <b>[ENTER]</b> .
6.	Mean= .8333333333 Standard Dev= .8333333333 f(2)= .1607510288 P(2)= .9645061728 Q(2)= .0354938272	Displays all requested values. Press <b>[ENTER]</b> after each output to proceed.
7.	Exit Program?	Press <b>[Y]</b> .

### Summary

There is a 16.1% chance of rolling exactly two fours in five rolls of a balanced die.

# The Chi-Square Distribution (CHI2) Program

---

This program computes probabilities based on the chi-square distribution.

---

## Method Used in the Program

When  $n$  is greater than 30, the following equation approximates the standard normal distribution.

$$z = \sqrt{2\chi^2} - \sqrt{2df}$$

If this approximation is not used for  $n > 30$ , the following inequality must be satisfied to prevent incorrect results.

$$e^{\chi^2/2} \leq 9.999999 \times 10^{99}$$

A series expansion is used to approximate  $Q(\chi^2)$ . When the degrees of freedom are odd and  $\chi = \sqrt{\chi^2}$ ,

$$Q(\chi^2; df) = 2Q(\chi) + 2F(\chi) \sum_{r=1}^{\frac{df-1}{2}} \frac{\chi^{2r-1}}{1 \times 3 \times 5 \dots (2r-1)}$$

where  $Q(\chi)$  and  $F(\chi)$  are computed using the Normal Distribution program. If the degrees of freedom are even,

$$Q(\chi^2; df) = e^{-m} \sum_{j=0}^{c-1} \frac{m^j}{j!}$$

where  $c = df/2$  and  $m = \chi^2/2$ . You can then calculate the following probability manually.

$$P(\chi^2) = 1 - Q(\chi^2) = \Pr(X \leq \chi^2)$$

## Reference

*Handbook of Mathematical Functions*, Abramowitz and Stegun. National Bureau of Standards, 1964, p. 941.

## Using the Chi-Square Distribution Program

To execute the Chi-Square Distribution program, type RUN "CHI2" and press [ENTER].

Step	Display	Procedure/Comment
1.	CHI-SQUARE DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter df:	Press [ENTER] to accept current value, or type the degrees of freedom and press [ENTER].
5.	Enter Chi-Square:	Press [ENTER] to accept current value, or type the chi-square value and press [ENTER].
6.	Q(###)=	Displays the area of the curve to the right of the chi-square value. Press [ENTER] to proceed.
7.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

## Application: Comparing Sample and Population Variances

In this example, you use the Chi Square Distribution program to compare the sample variance with the assumed population variance.

### Example

Twenty scales are tested for accuracy and yield a sample variance of 3 ounces. Use the formula  $\chi^2 = (ns^2/\sigma^2)$  to verify that this sample variance does not contradict the assumption that the population variance is less than 2.5 ounces at the 95% confidence level. (This formula is based on  $\chi^2$  having  $n - 1$  degrees of freedom.)

To execute the Chi-Square Distribution program, type **RUN "CHI2"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	CHI-SQUARE DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter df: 0	Type <b>19</b> and press <b>[ENTER]</b> .
4.	Enter Chi-Square: 0	Type <b>20*3/2.5</b> and press <b>[ENTER]</b> .
5.	$Q(24/19) = .1961523586$	Displays the area of the curve to the right of the chi-square value. Press <b>[ENTER]</b> to proceed.
6.	Exit Program?	Press <b>[Y]</b> .

### Summary

The program tests the hypothesis  $H_0: \sigma^2 \leq 2.5$  against  $H_1: \sigma^2 > 2.5$ . However, to complete the evaluation, you must use an upper-tailed test to test

$$H_0: \chi^2 \leq ns^2/\sigma^2 \text{ against } H_1: \chi^2 > ns^2/\sigma^2$$

If  $Q(\chi^2) \leq 0.05$  (as defined by the 95% confidence level), you must reject  $H_0$ . In this case,  $Q(\chi^2) = 0.1961524$ , and you fail to reject  $H_0$ .

# The F-Distribution (FDIS) Program

This program computes probabilities based on the F-distribution. The tail area of the appropriate curve is calculated from one of the three formulas that follow.

## Method Used in the Program

If  $df_1$  is even, the formula is

$$Q(F; df_1, df_2) = x^{\frac{df_2}{2}} \left( 1 + \frac{df_2}{2} (1-x) + \frac{df_2(df_2+2)}{2 \times 4} (1-x)^2 + \dots + \frac{df_2(df_2+2) \dots (df_2+df_1-4)}{2 \times 4 \dots (df_2-2)} (1-x)^{\frac{df_2-2}{2}} \right)$$

where  $x = df_2 / (df_2 + df_1 F)$ . If  $df_2$  is even, then

$$Q(F; df_1, df_2) = 1 - (1-x)^{\frac{df_1}{2}} \left( 1 + \frac{df_1}{2} x + \frac{df_1(df_1+2)}{2 \times 4} x^2 + \dots + \frac{df_1(df_1+2) \dots (df_2+df_1-4)}{2 \times 4 \dots (df_2-2)} x^{\frac{df_2-2}{2}} \right)$$

where  $x$  is defined above. When both  $df_1$  and  $df_2$  are odd, the formula is

$Q(F; df_1, df_2) = 1 - A(t; df_2) + \beta(df_1, df_2)$  where

$$A(t; df_2) = \begin{cases} \frac{2}{\pi} (\theta + \sin \theta (\cos \theta + \frac{2}{3} \cos^3 \theta + \dots \\ \frac{2\theta}{\pi} \text{ for } df_2 = 1 \\ \frac{2 \times 4 \dots (df_2-3)}{3 \times 5 \dots (df_2-2)} \cos^{df_2-2} \theta) \text{ for } df_2 > 1 \text{ and} \end{cases}$$

$$\beta(df_1, df_2) = \begin{cases} \frac{2}{\pi} \left( \frac{\frac{df_2-1}{2}!}{\left( \frac{df_2-2}{2}! \right)} \sin \theta \cos^{1/2} \theta \left( 1 + \frac{df_2+1}{3} \sin^2 \theta + \dots \right. \right. \\ \left. \left. 0 \text{ for } df_1 = 1 \right. \right. \\ \left. \left. + \frac{(df_2+1)(df_2+3) \dots (df_1+df_2-4) \sin^{df_1-3} \theta}{3 \times 5 \dots (df_1-2)} \right) \text{ for } df_1 > 1 \right\}$$

and  $\theta = \arctan \sqrt{\frac{df_1}{df_2} F}$

## Reference

*Handbook of Mathematical Functions*, Abramowitz and Stegun; National Bureau of Standards, 1964, p. 946.

## Using the F-Distribution Program

To execute the F-Distribution program, type RUN "FDIS" and press [ENTER].

Step	Display	Procedure/Comment
1.	F-DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter df1:	Press [ENTER] to accept current value, or type the degrees of freedom (numerator) and press [ENTER].
5.	Enter df2:	Press [ENTER] to accept current value, or type the degrees of freedom (denominator) and press [ENTER].
6.	Enter F:	Press [ENTER] to accept current value, or type the F-statistic and press [ENTER].
7.	Q(###)=	Displays the area of the curve to the right of the F-statistic. Press [ENTER] to proceed.
8.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

## Application: Verifying an F-Statistic

In this example, you use the F-Distribution program with an open-tailed test to determine whether you should reject your null hypothesis.

### Example

Suppose that you evaluated some data, using a one-way analysis of variance test, and found an F-statistic of  $F(2,24) = 2.63$ . Using an open-tailed test, determine whether you should reject your null hypothesis when testing at the 90% confidence level.

To execute the F-Distribution program, type **RUN "FDIS"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	F-DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter df1: 0	Type <b>2</b> and press <b>[ENTER]</b> .
4.	Enter df2: 0	Type <b>24</b> and press <b>[ENTER]</b> .
5.	Enter F: 0	Type <b>2.63</b> and press <b>[ENTER]</b> .
6.	$Q(2/24) = .0927348639$	Displays the area of the curve to the right of the F-statistic. Press <b>[ENTER]</b> to proceed.
7.	Exit Program?	Press <b>[Y]</b> .

### Summary

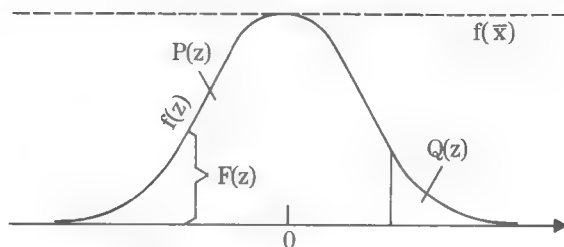
To test at the 90% confidence level,  $H_0$  should be rejected if  $P(F) > 0.90$ . This is the same as rejecting  $H_0$  when  $Q(F) \leq 0.10$ . Since  $Q(2.63) < 0.10$  for 2 and 24 degrees of freedom, you can reject  $H_0$ .

# The Normal Distribution (NORMAL) Program

The Normal Distribution program computes the standard deviate ( $z$ ), the height of the curve at  $z$ , or the area under the curve to the right of  $z$ .

## Method Used in the Program

The following graph summarizes the concepts covered by the Normal Distribution program.



The  $z$  value is computed with the equation

$$z = \frac{(x - \bar{x})}{\sigma}$$

where  $\bar{x}$  = population mean

$\sigma$  = standard deviation

$z$  = number of standard deviations that  $x$  is away from the mean.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-t^2/2} dt$$

$$= F(z)(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + \epsilon(z))$$

where  $\epsilon(z) < 7.5 \times 10^{-8}$

$$t = \frac{1}{1 + pz}$$

$$p = .2316419$$

$$b_1 = .319381530$$

$$b_2 = -.356563782$$

$$b_3 = 1.781477937$$

$$b_4 = -1.821255978$$

$$b_5 = 1.330274429$$



---

**Method Used in  
the Program  
(Continued)**

To compute  $z$ , given  $p = Q(z)$

$$z = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \varepsilon(p)$$

where

$$|\varepsilon(p)| < 4.5 \times 10^{-4}$$

$$t = \sqrt{\ln(1/p^2)}$$

$$c_0 = 2.515517 \quad d_1 = 1.432788$$

$$c_1 = .802853 \quad d_2 = .189269$$

$$c_2 = .010328 \quad d_3 = .001308$$

**Reference**

*Handbook of Mathematical Functions*, Abramowitz and Stegun. National Bureau of Standards, 1964, pp. 932-933.

## Using the Normal Distribution Program

To execute the Normal Distribution program, type RUN "NORMAL" and press [ENTER].

Step	Display	Procedure/Comment
1.	NORMAL DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	0:Menu 1:Q(z) 2:F(Z) 3:Z 4:Exit	Displays the menu, pausing while each line is displayed.
5.	Enter Option:	Press [ENTER] to accept current value, or type menu option number and press [ENTER]. 0: Repeat menu. Go to step 4. 1: Area of the normal curve to the right of the standard deviate. Go to step 6. 2: Height of the normal curve at the standard deviate. Go to step 8. 3: Standard deviate given area to the right of normal curve. Go to step 10. 4: Exit program.
6.	Enter z:	Press [ENTER] to accept current value, or type the standard deviate and press [ENTER].
7.	Q(###)=	Displays the area of the curve. Press [ENTER] to proceed. Go to step 5.
8.	Enter z:	Press [ENTER] to accept current value, or type standard deviate and press [ENTER].
9.	F(###)=	Displays the height of the normal curve at z. Press [ENTER] to proceed. Go to step 5.

---

Step	Display	Procedure/Comment
10.	Enter $Q(z)$ :	Press <b>[ENTER]</b> to accept current value, or type the area to the right of the normal curve and press <b>[ENTER]</b> .
11.	$z=$	Displays the standard deviate. Press <b>[ENTER]</b> to proceed. Go to step 5.

---

## Application: Calculating Probability (Normal Distribution)

In this example, you determine the fraction of a population that exceeds a specific criteria.

### Example

If the life of a flashlight battery is normally distributed with a mean of 120 hours and a standard deviation of 10 hours, find the probability that the battery will last for more than 100 hours.

To execute the Normal Distribution program, type **RUN "NORMAL"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	NORMAL DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	0:Menu 1:Q(z) 2:F(Z) 3:Z 4:Exit	Displays Menu.
4.	Enter Option: 0	Type <b>1</b> and press <b>[ENTER]</b> .
5.	Enter z: 0	Type <b>(100 - 120)/10</b> and press <b>[ENTER]</b> .
6.	Q(-2)= .977249938	Displays the area of the curve. Press <b>[ENTER]</b> to proceed.
7.	Enter Option: 1	Type <b>4</b> to exit program and press <b>[ENTER]</b> .

### Summary

In this example,  $x$  is first converted to a standard deviate using the expression

$$z = (x - \bar{x})/\sigma$$

When this conversion is made, the probability of the battery lasting more than 100 hours is found as

$$\Pr(Z > z) = Q(z).$$

# The Poisson Distribution (POISS) Program

This program computes probabilities based on the Poisson distribution.

## Method Used in the Program

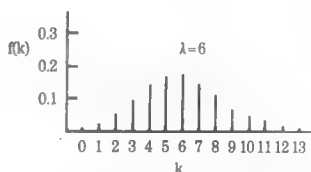
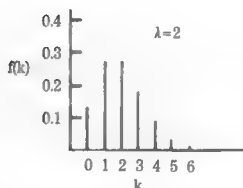
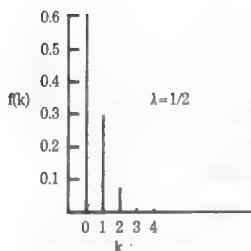
To use this program, you need to know the  $m$  parameter. The  $m$  parameter is the average number of successes ( $k$ ) which should occur in a sample, defined as:

$$m \text{ parameter} = \text{sample size} \times \text{probability of success}$$

which is also defined as:

$$m = \lambda \times t$$

where  $\lambda$  (lambda) represents the mean rate at which successes occur during the process and  $t$  represents the period of time. Using the second formula, the  $m$  parameter represents the mean number of successes ( $k$ ) which occur during time period  $t$ . Several Poisson distributions are illustrated below with various values for  $\lambda$ .



The probability of exactly  $k$  successes is labeled as  $f(k)$ .

$$f(k) = \frac{m^k e^{-m}}{k!} \quad \text{where } m = np$$

$k$  = the number of successes

## Reference

*Statistical Distributions*, Hastings and Peacock. Wiley and Sons, 1975, pp. 108–112.

## Using the Poisson Distribution Program

To execute the Poisson Distribution program, type RUN "POISS" and press [ENTER].

Step	Display	Procedure/Comment
1.	POISSON DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter m:	Press [ENTER] to accept current value, or type the m parameter and press [ENTER].
5.	Enter k:	Press [ENTER] to accept current value, or type the number of successes and press [ENTER].
6.	$f(k)=$	Displays the probability of exactly k successes. Press [ENTER] to proceed.
7.	$P(k)=$	Displays the probability of k or fewer successes. Press [ENTER] to proceed.
8.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

## Application 1: Calculating Probability (Poisson Distribution)

You can use this program to determine the probability of an occurrence.

### Example

The number of accidents that occur in a busy intersection during a week are observed. They follow a Poisson distribution with  $m = 2.5$ . What is the probability that there will be more than four accidents at this intersection during a given week?

Execute the Poisson Distribution program by entering **RUN "POISS"**.

Step	Display	Procedure/Comment
1.	POISSON DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	Press [N].
3.	Enter m: 0	Type 2.5 and press [ENTER].
4.	Enter k: 0	Type 4 and press [ENTER].
5.	f(4)= .1336018858	Displays the probability of exactly k successes. Press [ENTER] to proceed.
6.	P(4)= .8911780189	Displays the probability of k or fewer successes. Press [ENTER] to proceed.
7.	Exit Program?	Press [Y].

### Summary

$Q(k)$  is calculated by using the fact that  $Q(k) = 1 - P(k)$ . After exiting the program, type  $1 - .8911780189$  and press [ENTER] to calculate  $Q(k) = .1088219811$ , which gives the probability of more than four accidents per week (10.9%).

## Application 2: Approximating the Binomial Distribution

---

When  $n$  is large, you can use the Poisson distribution to approximate the binomial distribution.

---

### Example

Find the probability of obtaining 4 successes in 100 independent trials when  $p = 0.04$ . The  $m$  parameter is

$m = \text{sample size} \times \text{probability of success}$

To execute the Poisson Distribution program, type **RUN** "POISS" and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	POISSON DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter m: 0	Type <b>100*.04</b> and press <b>[ENTER]</b> .
4.	Enter k: 0	Type <b>4</b> and press <b>[ENTER]</b> .
5.	f(4)= .1953668148	Displays the probability of exactly $k$ successes. Press <b>[ENTER]</b> to proceed.
6.	P(4)= .6288369352	Displays the probability of $k$ or fewer successes. Press <b>[ENTER]</b> to proceed.
7.	Exit Program?	Press <b>[Y]</b> .

### Summary

The estimated probability is 0.195. This approximation shows close agreement with the binomial distribution, where  $f(k) = 0.199$ .



# The Student's t-Distribution (STUDENT) Program

---

This program computes probabilities based on the Student's t-distribution.

---

## Inputs for the Program

When using this program, you must know the degrees of freedom for your sample, which must be at least  $df = 2$ . You must also know the t-statistic, which can be computed by either the t-Test: Paired or t-Test: Unpaired program.

## Method Used in the Program

Because the Student's t-Distribution has a relationship to the F-distribution, the value  $P(t)$  is calculated from the following formula.

$$\Pr[(T:df) \leq x] = \frac{1}{2} \{1 + \Pr[(F:1, df) \leq x^2]\}$$

Using this result, you may calculate the following probabilities manually.

$$Q(t) = 1 - P(t) = \Pr(T > t)$$

$$A(t) = 2P(t) - 1 = \Pr(T \leq |t|)$$

where  $A(t)$  is the acceptance region when using a two-tailed test.

## Reference

*Statistical Distributions*, Hastings & Peacock. Wiley, 1975, p. 120.

## Using the Student's t-Distribution Program

---

To execute the Student's t-Distribution program, type RUN "STUDENT" and press [ENTER].

---

Step	Display	Procedure/Comment
1.	STUDENT'S t-DISTRIBUTION	Wait while the program name is displayed, or press [ENTER] to continue.
2.	Use Printer?	a. If you want to use the printer, press [Y]. b. If you do not want to use the printer, press [N]. Go to step 4.
3.	Enter Device Code:	Type output device code and press [ENTER].
4.	Enter df:	Press [ENTER] to accept current value, or type the degrees of freedom and press [ENTER].
5.	Enter t:	Press [ENTER] to accept current value, or type the t-statistic and press [ENTER].
6.	P(###)=	Displays the area of the curve to the left of the t-statistic. Press [ENTER] to proceed.
7.	Exit Program?	a. To exit program, press [Y]. b. To continue program, press [N]. Go to step 4.

---

## Application 1: Testing the Uniformity of Engine Performance

---

The following example uses the Student's t-Distribution program to determine if there has been a significant change in gasoline consumption for an experimental engine.

---

### Example

An experimental engine has been previously tested for gasoline consumption. A short time later, it is decided to test the engine again to see if it is still operating at the same average gasoline consumption rate. Eight tests are run, and a t-statistic of 2.30 is obtained by comparing the new average consumption with the old average consumption. At a 95% confidence level, test the null hypothesis that the engine is operating at the same consumption rate.

To execute the Student's t-distribution program, type **RUN "STUDENT"** and press **[ENTER]**.

Step	Display	Procedure/Comment
1.	STUDENT'S t-DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter df: 0	Type <b>7</b> and press <b>[ENTER]</b> .
4.	Enter t: 0	Type <b>2.3</b> and press <b>[ENTER]</b> .
5.	P(2.3)= .9725044524	Displays the area of the curve to the left of the t-statistic. Press <b>[ENTER]</b> to proceed.
6.	Exit Program?	Press <b>[Y]</b> .

### Summary

You use a two-tailed test for this example, since the new consumption rate may be significantly greater or significantly less than the old consumption rate. As a result, you fail to reject the null hypothesis if  $0.025 \leq P(t) \leq 0.975$ . Since  $P(t) < .975$ , you fail to reject your null hypothesis.

## Application 2: Finding the Area Under the t-Curve

---

This example uses the Student's t-distribution to compute the area under the t-curve and compare this area with the same area for the standard normal curve.

---

### Example

Find the area under the t curve with 30 degrees of freedom between  $t=0$  and  $t=1.2$ . Compare this area with the corresponding area for the standard normal curve.

To execute the Student's t-distribution program, type **RUN "STUDENT"** and press **[ENTER]**.

---

Step	Display	Procedure/Comment
1.	STUDENT'S t-DISTRIBUTION	Wait while the program name is displayed, or press <b>[ENTER]</b> to continue.
2.	Use Printer?	Press <b>[N]</b> .
3.	Enter df: 0	Type <b>30</b> and press <b>[ENTER]</b> .
4.	Enter t: 0	Type <b>1.2</b> and press <b>[ENTER]</b> .
5.	$P(1.2) = .8802348246$	Displays the area of the curve to the left of the t-statistic. Press <b>[ENTER]</b> to proceed.
6.	Exit Program?	Press <b>[N]</b> .
7.	Enter df: 30.	Press <b>[ENTER]</b> to accept current value.
8.	Enter t: 1.2	Type <b>0</b> and press <b>[CTL][+][ENTER]</b> .

---

## Application 2: Finding the Area Under the t-Curve (Continued)

Step	Display	Procedure/Comment
9.	$P(0) = .5$	Displays the area of the curve to the left of the t-statistic. Press <b>[ENTER]</b> to proceed.
10.	Exit Program?	Press <b>[Y]</b> .

### Summary

This area is found by performing the following calculations:

$$\begin{aligned}P(t \leq 1.2) - P(t \leq 0) &= P(1.2) - P(0) \\&= .8802348246 - .5 \\&= .3802348264\end{aligned}$$

Executing the same calculations with the Normal Distribution program (using  $P(z) = 1 - Q(z)$ ) results in Area = .3849302679. Note that these areas are approximately the same, demonstrating the fact that the t-curve becomes the standard normal curve for  $df \geq 30$ .

# Appendix A: User-Accessible Subprograms

---

This appendix describes the subprograms used by the Statistics Library programs.

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# Subprogram Information

---

Many of the routines used in the programs of this library are developed as independent subprograms. These subprograms may be accessed for use in developing your own BASIC programs.

---

## The Information in this Appendix

This appendix provides the information necessary for including the cartridge subprograms in your own BASIC programs. The subprograms are arranged in alphabetical order by their calling names. The discussion of each subprogram includes:

- ▶ **Format**—illustrates the format necessary for calling the subprogram and identifies the requirements, restrictions and purpose of each element in the argument list. Elements in all capital letters should be entered exactly as they appear in the format line.
- ▶ **Example**—demonstrates the procedure for executing the subprogram.

## Calling a Subprogram

To call the subprogram, you use the CALL statement with the name of the subprogram and its assigned arguments. For example, to call the subprogram NORM at line number 230 in your program, enter

**230 CALL NORM(*z*,*q*)**

where ***z*** passes the value for the standard deviate, and ***q*** returns the area to the right of the standard deviate.

Refer to the *TI-74 Programming Reference Guide* for details on the restrictions and requirements for using subprograms.

## BI and CHI

---

The BI subprogram finds the probabilities associated with  $n$  trials of an experiment. The CHI subprogram finds the area to the right of the Chi-square statistic.

---

### BI Format

CALL BI( $n,p,k,f,pr$ )

$n$  Passes the number of trials to be attempted.

$p$  Passes the probability of successes desired.

$k$  Passes the number of successes.

$f$  Returns the probability of exactly  $k$  successes.

$pr$  Returns the probability of  $k$  or fewer successes.

### Example

```
100 N=50
110 P=.5
120 K=28
130 CALL BI (N,P,K,F,PR)
```

F returns a value of .0788256707 and PR returns a value of .8388818398.

### CHI Format

CALL CHI( $degrees,chisqr,output$ )

$degrees$  Passes the degrees of freedom.

$chisqr$  Passes the chi-square statistic.

$output$  Returns the area to the right of the chi-square statistic.

### Example

```
100 DEGREES=19
110 CHISQR=24
120 CALL CHI (DEGREES,CHISQR,OP)
```

OP returns a value of .1961523586.



## F and IR

---

The F subprogram finds the height of the normal curve at the standard deviate. The IR subprogram inputs a number.

---

### F Format

CALL F(*z*,*f*)

*z* Passes the value for the standard deviate.

*f* Passes a dummy variable that returns the height of the normal curve at the standard deviate.

### Example

```
100 Z = .12
110 CALL F(Z,F)
```

F returns a value of .3960802118.

### IR Format

CALL IR(*tabpos*,*prompt\$*,*variable*,*device*)

*tabpos* Specifies tab position when printing the input value (only used with a printer).

*prompt\$* Passes a prompt describing the requested variable.

*variable* Returns the value entered.

*device* Passes the output device number.

### Example

```
100 CALL IR(15,"Standard Dev.",SD,PN)
```

1. When the line is executed, the subprogram asks for input with the prompt:

Enter Standard Dev.: X

where X is the current value of SD.

2. After input, the value of PN is checked. If it is 0, control returns to the calling program. If  $PN > 0$ , the prompt and the entered number are printed on device #PN:

Standard Dev. = X

Control then returns to the calling program.

## IZ and MN

---

The IZ program checks to see if the value is an integer. The MN subprogram finds the means and moments for paired or unpaired data in the form of one-dimensional arrays.

---

### IZ Format

CALL IZ(*variable*,*flag*)

*variable* Passes number to be validated.

*flag* Returns a value indicating the status of *variable*. If *variable* is a non-integer, *flag* is set to -1, and the message VALUE MUST BE AN INTEGER is printed. If *variable* is an integer, *flag* is set to 0.

### Example

100 CALL IZ(2.5,FLAG)

Prints VALUE MUST BE AN INTEGER. Sets FLAG to -1.

### MN Format

CALL MN(*x()*,*f()*,*k*,*n*,*a*,*h*,*g*,*m2*,*m3*,*m4*,*f1*,*f2*,*total*)

*x()* Passes the values in the one-dimensional array.

*f()* Passes the values for the frequencies of the X values.

- ▶ If the data is ungrouped, passes  $f(0) = 0$ .
- ▶ If the data is grouped, passes  $f(0) = -2$ .

*k* Passes the calculation set to be performed.

- ▶ For a single set of input,  $k = 0$  selects calculations for final output.
- ▶ For sequential sets of input,  $k = 0$  selects calculations for the first set of input.
- ▶ For sequential sets of input,  $k = 1$  selects calculations for the intermediate sets of input.
- ▶ For sequential sets of input,  $k = -1$  selects calculations for final output.

*n* Passes the number of groups in the data set.

*a* Returns the arithmetic mean if  $k$  is 0 or -1.

*h* Returns the harmonic mean if  $k$  is 0 or -1.

*g* Returns the geometric mean if  $k$  is 0 or -1.

**MN Format  
(Continued)**

---

<i>m2</i>	Returns the second moment if <i>k</i> is 0 or -1.
<i>m3</i>	Returns the third moment if <i>k</i> is 0 or -1.
<i>m4</i>	Returns the fourth moment if <i>k</i> is 0 or -1.
<i>f1</i>	Returns -1 as flag if invalid numbers are used to calculate harmonic mean. Otherwise returns 0.
<i>f2</i>	Returns -1 as flag if invalid numbers are used to calculate geometric mean. Otherwise returns 0.
<i>total</i>	Returns the total number of observations in the final set of input if <i>k</i> is 1 or -1.

**Example**

```
100 DIM X(4),F(4)
110 DATA 3,4,5,6
120 DATA 1,2,2,1
130 RESTORE 110
140 FOR I=1 to 4:READ X(I):NEXT I
150 RESTORE 120
160 FOR I=1 to 4:READ F(I):NEXT I
170 CALL MN(X(),F(),-1,4,AMEAN,GMEAN,HMEAN,
      MOM2,MOM3,MOM4,FH,FG,TOTAL)
```

Returns the three means and three moments. Returns 0 for both flags and 6 for the value of TOTAL.

## NORM and POIS

---

The NORM subprogram finds the area to the right of the standard deviate for the normal distribution. The POIS subprogram finds the probabilities associated with the Poisson distribution.

---

**NORM Format**      `CALL NORM(z,q)`

*z*   Passes the value for the standard deviate.

*q*   Returns the area to the right of the standard deviate.

**Example**

```
100 Z=-2
```

```
110 CALL NORM(Z,Q)
```

Q returns a value of .977249938.

**POIS Format**      `CALL POIS(m,k,f,p)`

*m*   Passes the m parameter.

*k*   Passes the number of successes.

*f*   Returns the probability of exactly k successes.

*p*   Returns the probability of k or fewer successes.

**Example**

```
100 M=2.5
```

```
110 K=4
```

```
120 CALL POIS(M,K,F,P)
```

F returns a value of .1336018858 and P returns a value of .8911780189.

## SM and SORT

---

The SM subprogram finds the sum and the sum of squares for a row in a two-dimensional array. The SORT subprogram sorts a one-dimensional array in ascending order, using the Shell Sort method.

---

### SM Format

CALL SM(*x*(,),*r*,*col*,*sum*,*sumsq*)

*x*(,) Passes the values for a two-dimensional array.

*r* Passes the number of the row to be summed.

*col* Passes the column position of the last element the summation is to include.

*sum* Returns the sum of the elements in the given row.

*sumsq* Returns the sum of the squares of the elements in the given row.

### SORT Format

CALL SORT(*x*( ),*n*)

*x*( ) Passes the elements of a one-dimensional array *x*( ) and returns the sorted array.

*n* Passes the number of elements in the array.

### Example

```
100 X(1)=4
110 X(2)=1
120 X(3)=3
130 X(4)=2
140 N=4
150 CALL SORT(X( ),N)
```

X( ) returns as

```
X(1)=1
X(2)=2
X(3)=3
X(4)=4
```

## UP and VAR

---

The UP subprogram prompts you for printer use. The VAR subprogram finds the area to the right of the F-statistic for the F-distribution.

---

### UP Format

CALL UP(*name\$*,*device*)

*name\$* Passes the name of program to be displayed.

*device* Returns the device number of the opened output device. 0 indicates output appears in the display. 1 indicates that the requested output device is opened as device #1.

### Example

```
100 CALL UP("NORMAL DISTRIBUTION",PN)
```

1. When the line is executed, NORMAL DISTRIBUTION appears in the display.
2. The subprogram then prompts: Use Printer?
  - Responding with [N] exits the subprogram.
  - Responding with [Y] causes the subprogram to prompt: Enter Device Code:
3. Enter the device code for the printer. PN is set to the device code for the printer, the device is opened for output, and control returns to the calling program.

### VAR Format

CALL VAR(*df1*,*df2*,*f*,*q*)

*df1* Passes the value for degrees of freedom (numerator).

*df2* Passes the value for degrees of freedom (denominator).

*f* Passes the value for F-statistic.

*q* Returns the area to the right of the F-statistic.

### Example

```
100 DF1=2
110 DF2=24
120 F=2.63
130 CALL VAR(DF1,DF2,F,Q)
```

Q returns a value of .0927348639.

## WR and YN

---

The WR subprogram prints a space and then prints a message.  
The YN subprogram asks a yes-or-no question.

---

### WR Format

CALL WR(*message\$,printer,pause-time*)

*message\$* Passes the message, in quotes, to be printed.

*printer* Passes a number to the output device where the message is to be printed. If *printer*=0, the message is displayed rather than printed.

*pause-time* Specifies the number of seconds to pause.

### Example

```
100 CALL WR("MESSAGE",PN,1)
```

When the line is executed, the message MESSAGE is displayed for one second and then printed.

### YN Format

CALL YN(*question\$,answer,device*)

*question\$* Passes the question to be asked. (Do not include a question mark; it is added automatically.)

*answer* Returns *answer*=0 when [N] is pressed, or returns *answer*=-1 when [Y] is pressed.

*device* Passes device number. If device number is zero, output is displayed. If the device number is greater than zero, *question\$* is printed on the specified device.

### Example

```
100 CALL YN("Edit",D,PN)
```

When the line is executed, the question Edit? appears in the display.

- ▶ Responding with [Y] sets D=-1 and prints the question on device #PN.
- ▶ Responding with [N] sets D=0. Control is then passed back to the calling routine.

## Z

---

Given an area under the normal curve, the Z subprogram finds the standard deviate.

---

### Z Format

CALL Z( $q$ , $z$ )

$q$  Passes the area under the normal curve.

$z$  Returns the standard deviate.

### Example

```
100 Q=.97725  
110 CALL Z(Q,DEV)
```

DEV returns a value of  $-2.000437772$ .



# Appendix B: Service and Warranty Information

---

This appendix describes the service provided by Texas Instruments and the warranty for the cartridge.

---

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## Service Information

---

If you experience a problem with the cartridge, you can call or write Consumer Relations to discuss the problem.

---

### For Service and General Information

If you have questions about service or the general use of the cartridge, please call Consumer Relations **toll-free** within the United States at:

**1-800-TI CARES (842-2737).**

From outside the United States, call 1-806-741-4800. (We cannot accept collect calls at this number.)

You may also write to the following address:

Texas Instruments Incorporated  
Consumer Relations  
P.O. Box 53  
Lubbock, Texas 79408

Please contact Consumer Relations:

- ▶ Before returning the cartridge for service
- ▶ For general information about using the cartridge

### For Technical Information

If you have technical questions about cartridge operation or programming applications, call 1-806-741-2663. We regret that we cannot accept collect calls at this number. As an alternative, you can write Consumer Relations at the address given above.

### Express Service

Texas Instruments offers an express service option for fast return delivery. Please call Consumer Relations at 1-800-TI CARES (842-2737) for information.

### Accessories

If you are unable to purchase calculator accessories (such as the CI-7 cassette interface cable) from your local dealer, you may order them from Texas Instruments. Please call Consumer Relations at 1-800-TI CARES (842-2737) for information.

---

### **Returning a Cartridge for Service**

A defective cartridge will be either repaired or replaced with the same or comparable reconditioned model (at TI's option) when it is returned, postage prepaid, to a Texas Instruments Service Facility.

Texas Instruments cannot assume responsibility for loss or damage during incoming shipment. For your protection, carefully package the cartridge for shipment and insure it with the carrier. Be sure to enclose the following items with the cartridge:

- ▶ Your full return address
- ▶ Any accessories related to the problem
- ▶ A note describing the problem you experienced
- ▶ A copy of your sales receipt or other proof of purchase to determine warranty status

Please ship the cartridge postage prepaid; COD shipments cannot be accepted.

### **In-Warranty Repair**

For a cartridge covered under the warranty period, no charge is made for service.

### **Out-of-Warranty Repair**

For an out-of-warranty cartridge, a flat-rate fee by model is charged for service. Estimates are not provided prior to repair; to obtain the service charge for a particular model, please call Consumer Relations before returning the cartridge to the Service Facility.

## Service Information (Continued)

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<b>Texas Instruments Service Facilities</b>	<b>U.S. Residents (U.S. Postal Service)</b>	<b>U.S. Residents (other carriers)</b>
	Texas Instruments	Texas Instruments
	P.O. Box 2500 Lubbock, Texas 79408	2305 N. University Lubbock, Texas 79415
	<b>Canadian Residents Only</b>	
	Texas Instruments	
	41 Shelley Road	
	Richmond Hill, Ontario L4C 5G4	

# One-Year Limited Warranty

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**This Texas Instruments software cartridge warranty extends to the original consumer purchaser of the product.**

---

## **Warranty Duration**

This cartridge is warranted to the original consumer purchaser for a period of one (1) year from the original purchase date.

## **Warranty Coverage**

This cartridge is warranted against defective materials and construction. This warranty covers the electronic and case components of the software cartridge. These components include all semiconductor chips and devices, plastics, boards, wiring, and all other hardware contained in this cartridge ("the Hardware"). This limited warranty does not extend to the programs contained in the cartridge and the accompanying book materials ("the Programs"). **The warranty is void if the cartridge has been damaged by accident or unreasonable use, neglect, improper service, or other causes not arising out of defects in materials or construction.**

## **Warranty Disclaimers**

**Any implied warranties arising out of this sale, including but not limited to the implied warranties of merchantability and fitness for a particular purpose, are limited in duration to the above one year period. Texas Instruments shall not be liable for loss of use of the cartridge or other incidental or consequential costs, expenses, or damages incurred by the consumer or any other user.**

Some states do not allow the exclusion or limitations of implied warranties or consequential damages, so the above limitations or exclusions may not apply to you.

## **Legal Remedies**

This warranty gives you specific legal rights, and you may also have other rights that vary from state to state.

## **Warranty Performance**

During the above one-year warranty period, your TI cartridge will be either repaired or replaced with a reconditioned comparable model (at TI's option) when the cartridge is returned, postage prepaid, to a Texas Instruments Service Facility. The repaired or replacement cartridge will be in warranty for the remainder of the original warranty period or for six months, whichever is longer. Other than the postage requirement, no charge will be made for such repair or replacement. Texas Instruments strongly recommends that you insure the product for value, prior to mailing.



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1060305-0101